09365060

Principles of Automatic Control (1)

自动控制原理1 （全英语教学示范课程）

Topic 8

Frequency Response Techniques

(Chapter 10 in text book)

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New terminologies in this topic

- Frequency response 频率响应
- Magnitude frequency response 幅频响应
- Phase frequency response 相频响应
- Decibel 分贝
- Bode Plot 波特图
- Nyquist diagram 奈奎斯特图
- Nyquist criterion 奈奎斯特判据
- Approximation 近似
- Low-frequency asymptote 低频渐进线
- High-frequency asymptote 高频渐进线
- Break frequency 转折频率
- Normalize 正规化
- Scale 比例
- Contour 周线（闭合曲线）
- Parenthetical 括号里的
- Resultant 结果
- Emanate 放射
- Detour 绕道
- Mapping 映射

Learning Outcomes for Topic 8

After completing this topic, you will be able to:

- Define and plot the frequency response of a system;
- Plot asymptotic approximations to the frequency response of a system;
- Sketch a Nyquist diagram;
- Use the Nyquist criterion to determine the stability of a system；
- Find stability and gain and phase margins using Nyquist diagrams and Bode plots；
- Find the bandwidth, peak magnitude, and peak frequency of a closed-loop frequency response given the closed-loop time response parameters of peak time, settling time, and percent overshoot.

Outline

- Brief Introduction
- Asymptotic Approximations: Bode Plots
- Introduction to the Nyquist Criterion
- Sketching the Nyquist Diagram
- Stability via the Nyquist Diagram
- Gain Margin and Phase Margin via the Nyquist Diagram
- Stability, Gain Margin, and Phase Margin via Bode Plots
- Relation Between Closed-Loop Transient and Closed-Loop Frequency Responses
- Summary

1. Brief Introduction

- Frequency response methods are older than the root locus method.
- Frequency response yields a new vantage point from which to view feedback control systems.
- This technique has distinct advantages in the following situations:
  - When modeling transfer functions from physical data
  - When designing lead compensators to meet a steady-state error requirement and a transient response requirement
  - When finding the stability of nonlinear systems
  - In settling ambiguities when sketching a root locus

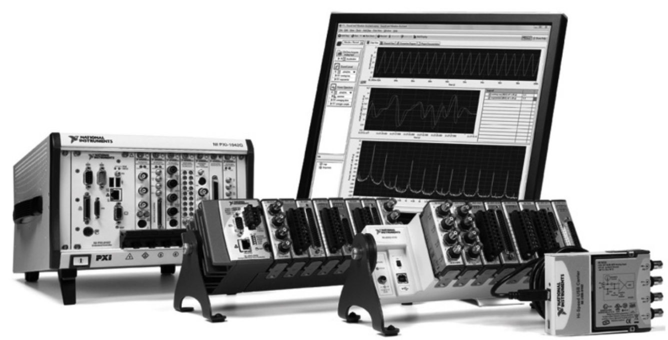
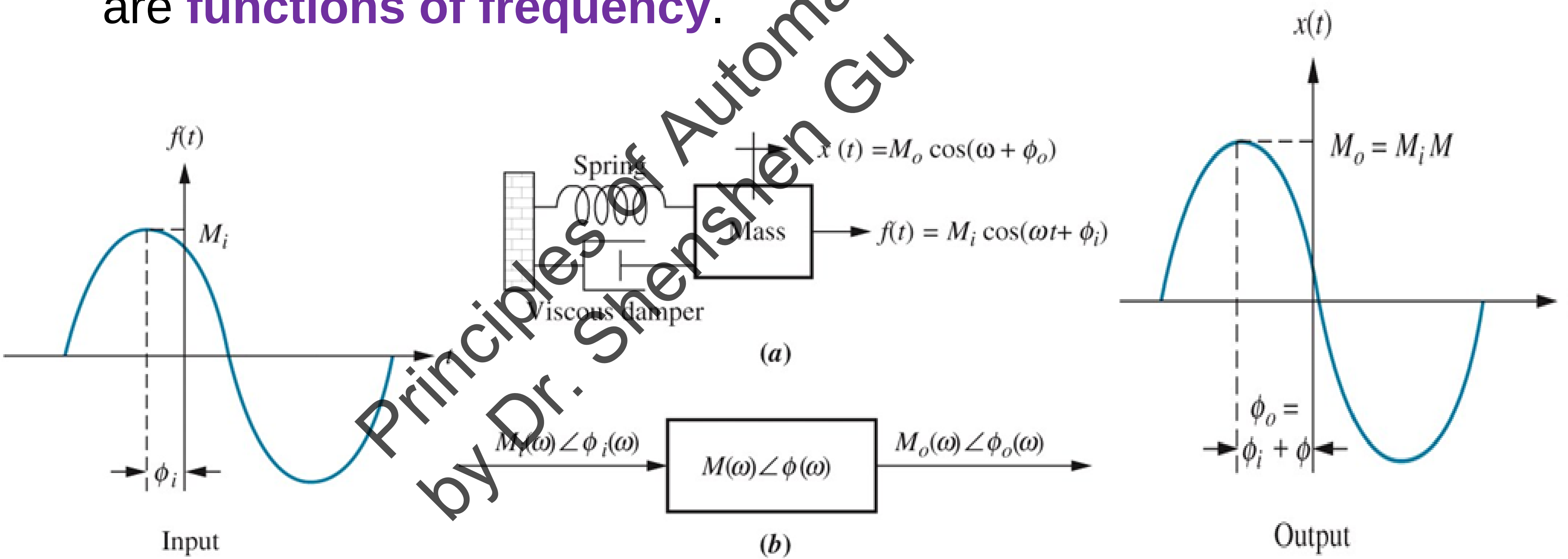


Figure 10.1
Courtesy National Instruments ©2010

The Concept of Frequency Response

- In the steady state, sinusoidal inputs to a linear system generate sinusoidal responses of the same frequency. Even though these responses are of the same frequency as the input, they differ in **amplitude** and **phase angle** from the input. These differences are **functions of frequency**.



- Assume that the system is represented by the complex number, *M(ω) ∠Φ(ω)*. The output steady-state sinusoid is found by multiplying the complex number representation of the input by the complex number representation of the system. Thus, the steady-state output sinusoid is

$$M_o(\omega)\angle\phi_o(\omega) = M_i(\omega)M(\omega)\angle\left[\phi_i(\omega)+\phi(\omega)\right]$$

- We see that the system function is given by

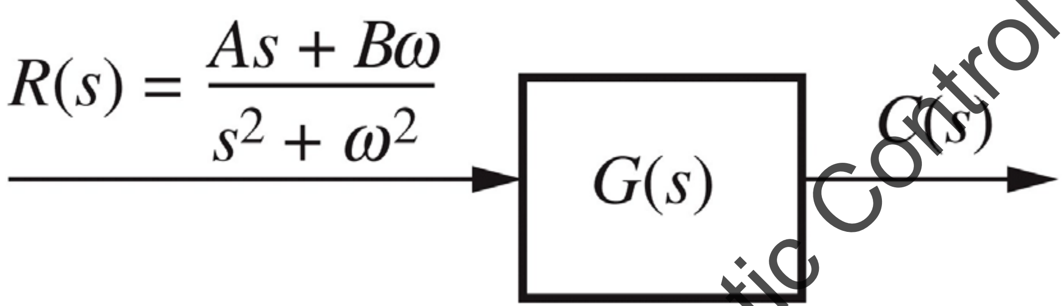
$$M(\omega) = \frac{M_o(\omega)}{M_i(\omega)} \qquad \phi(\omega) = \angle\phi_o(\omega) - \phi_i(\omega)$$

$M(\omega)$ Magnitude frequency response

$\phi(\omega)$ Phase frequency response

$M(\omega)\angle\phi(\omega)$ Frequency response

Analytical Expressions for Frequency Response



$$r(t)=A\cos\omega t+B\sin\omega t$$
$$=\sqrt{A^2+B^2}\cos\left[\omega t-\tan^{-1}(B/A)\right]$$
$$M_i\angle\phi_i$$
$$A-jB$$
$$M_i e^{j\phi_i}$$

$$C(s)=\frac{As+B\omega}{(s+j\omega)(s-j\omega)}G(s)$$
$$=\frac{K_1}{s+j\omega}+\frac{K_2}{s-j\omega}+\text{Partial fraction terms from } G(s)$$
$$K_1=\frac{As+B\omega}{s-j\omega}G(s)\Big|_{s\to -j\omega},\quad K_2=\frac{As+B\omega}{s+j\omega}G(s)\Big|_{s\to +j\omega}$$

$$C_{ss}(s)=\frac{K_1}{s+j\omega}+\frac{K_2}{s-j\omega}\qquad c(t)=M_i\left|G(j\omega)\right|\cos\left(\omega t+\phi_i+\angle G(j\omega)\right)$$

The frequency response of a system whose transfer function is *G(s)* is $G(j\omega)=G(s)\big|_{s\to j\omega}$

Plotting Frequency Response

- $G(j\omega)=M_G(\omega)\ \angle\Phi_G(\omega)$ can be plotted in several ways; two of them are
  - (1) as a function of frequency, with separate magnitude and phase plots;
  - (2) as a polar plot, where the phasor length is the magnitude and the phasor angle is the phase.
- When plotting separate magnitude and phase plots, the magnitude curve can be plotted in decibels (dB) vs. log ω, where dB=$20\log M$. The phase curve is plotted as phase angle vs. log ω. The motivation for these plots is shown in next section.

Example 10.1

Frequency Response from The Transfer Function

PROBLEM: Find the analytical expression for the magnitude frequency response and the phase frequency response for a system $G(s) = 1/(s+2)$. Also, plot both the separate magnitude and phase diagrams and the polar plot.

SOLUTION: First substitute $s = j\omega$ in the system function and obtain $G(j\omega) = 1/(j\omega + 2) = (2 - j\omega)/(\omega^2 + 4)$. The magnitude of this complex number, $|G(j\omega)| = M(\omega) = 1/\sqrt{(\omega^2 + 4)}$, is the magnitude frequency response. The phase angle of $G(j\omega)$, $\phi(\omega) = -\tan^{-1}(\omega/2)$, is the phase frequency response.

$G(j\omega)$ can be plotted in two ways: (1) in separate magnitude and phase plots and (2) in a polar plot. Figure 10.4 shows separate magnitude and phase diagrams, where the magnitude diagram is $20\log M(\omega) = 20\log(1/\sqrt{\omega^2+4})$ vs. $\log\omega$, and the phase diagram is $\phi(\omega) = -\tan^{-1}(\omega/2)$ vs. $\log\omega$. The polar plot, shown in Figure 10.5, is a plot of $M(\omega) < \phi(\omega) = 1/\sqrt{\omega^2+4} < -\tan^{-1}(\omega/2)$ for different ω.

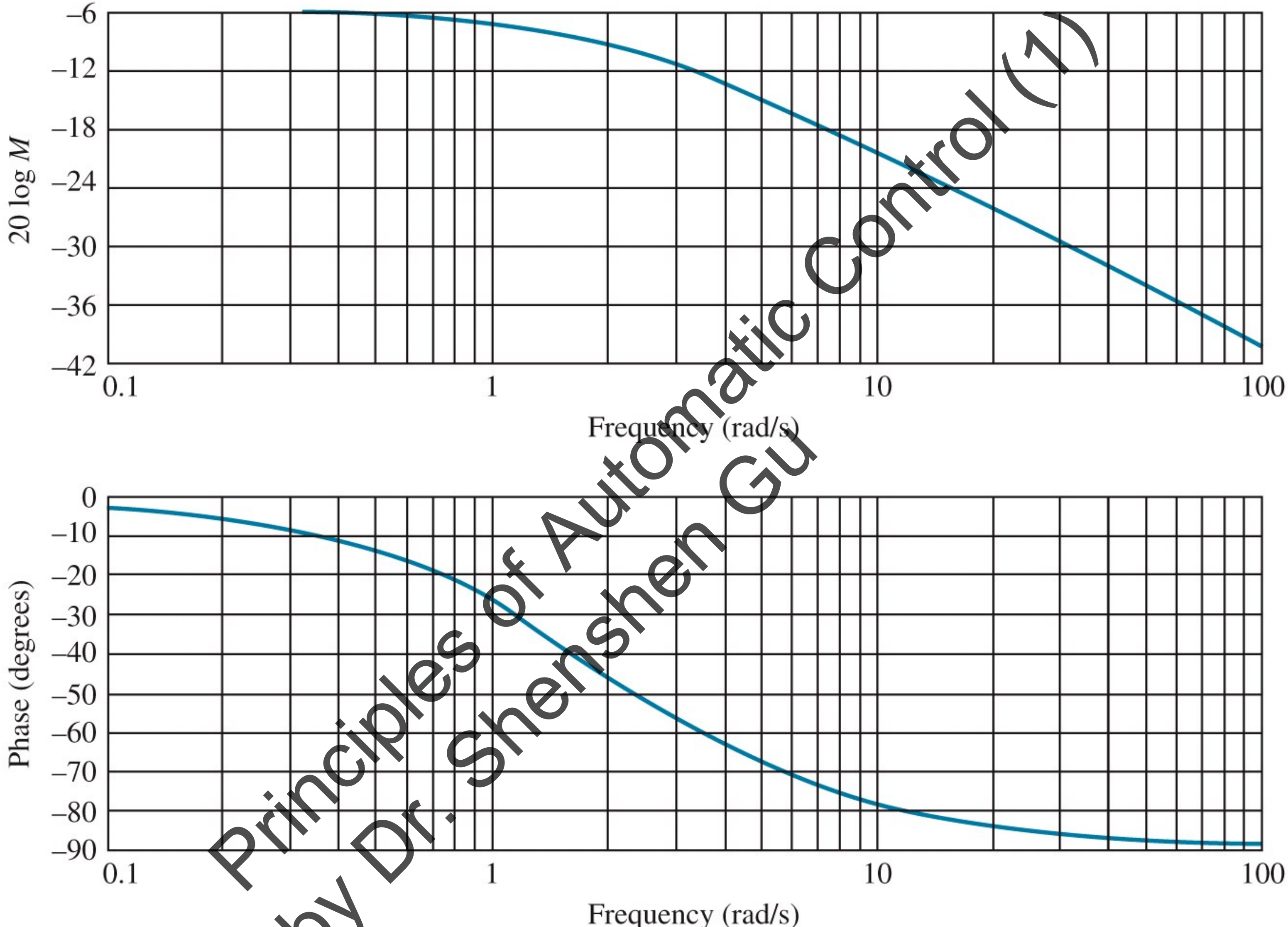

–6
–12
–18
–24
–30
–36
–42
20 log M
0.1
1
10
100
Frequency (rad/s)
0
–10
–20
–30
–40
–50
–60
–70
–80
–90
Phase (degrees)
0.1
1
10
100
Frequency (rad/s)

Figure 10.4

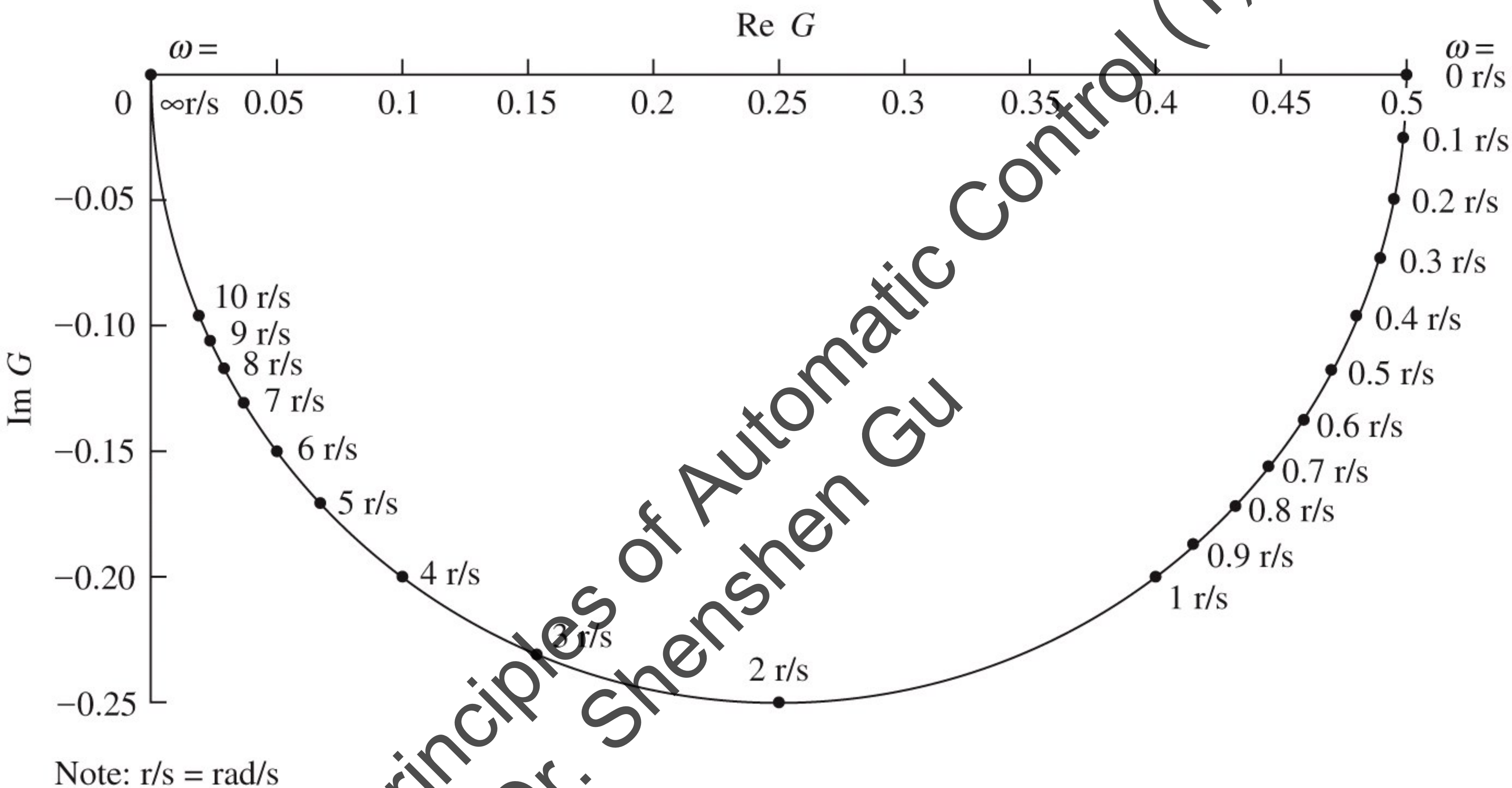

Re G
ω =
∞r/s
ω =
0 r/s
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
−0.05
−0.10
−0.15
−0.20
−0.25
Im G
0.1 r/s
0.2 r/s
0.3 r/s
0.4 r/s
0.5 r/s
0.6 r/s
0.7 r/s
0.8 r/s
0.9 r/s
1 r/s
2 r/s
3 r/s
4 r/s
5 r/s
6 r/s
7 r/s
8 r/s
9 r/s
10 r/s
Note: r/s = rad/s

Figure 10.5

Skill-Assessment Exercise 10.1

PROBLEM:

a. Find analytical expressions for the magnitude and phase responses of

$$G(s) = \frac{1}{(s+2)(s+4)}$$

b. Make plots of the log-magnitude and the phase, using log-frequency in rad/s as the ordinate.

c. Make a polar plot of the frequency response.

ANSWERS:

a. $M(\omega) = \frac{1}{\sqrt{(8-\omega^2)^2 + (6\omega)^2}}$; for $\omega \le \sqrt{8}$: $\phi(\omega) = -\arctan\left(\frac{6\omega}{8-\omega^2}\right)$, for $\omega > \sqrt{8}$: $\phi(\omega) = -\left[\pi + \arctan\left(\frac{6\omega}{8-\omega^2}\right)\right]$

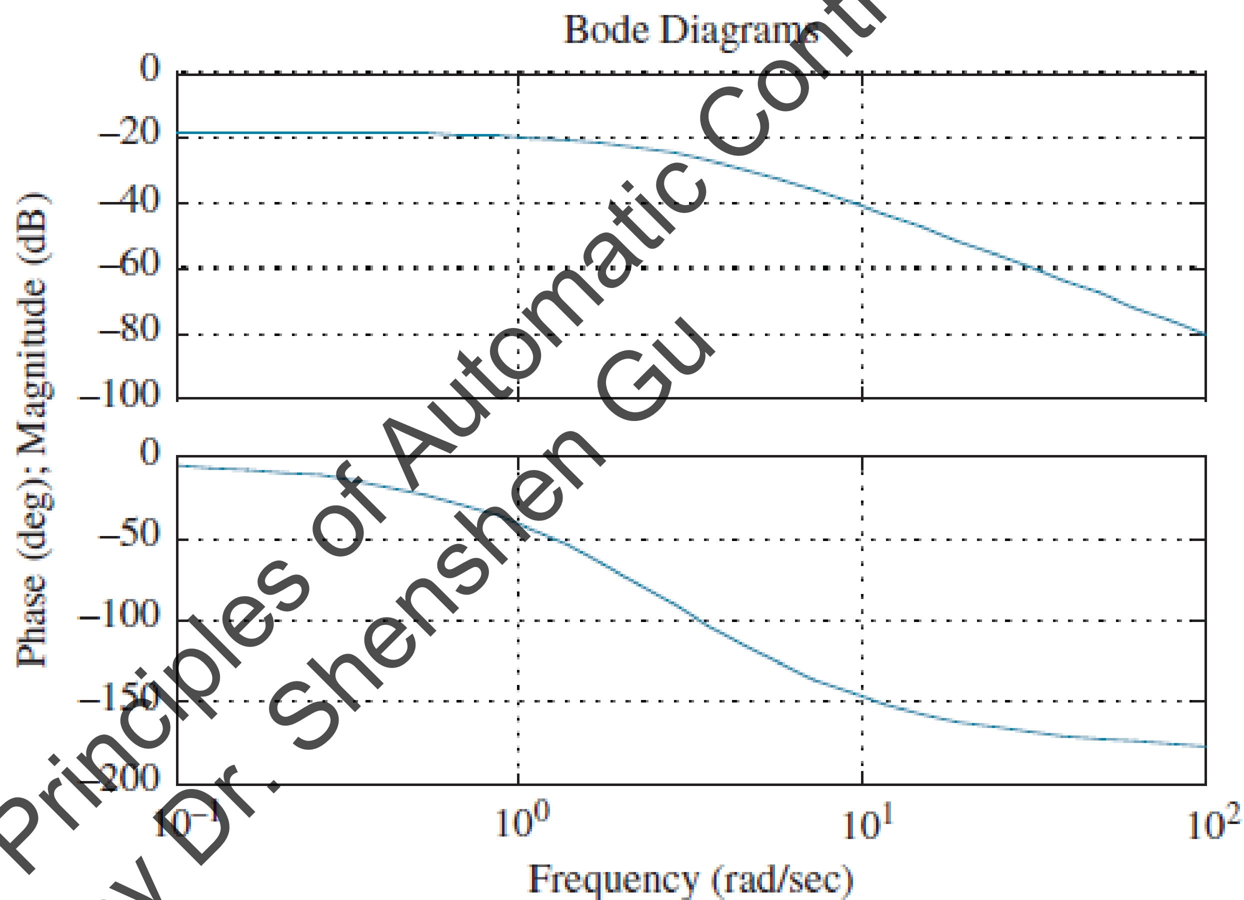
b. See the answer at www.wiley.com/college/nise.

c. See the answer at www.wiley.com/college/nise.

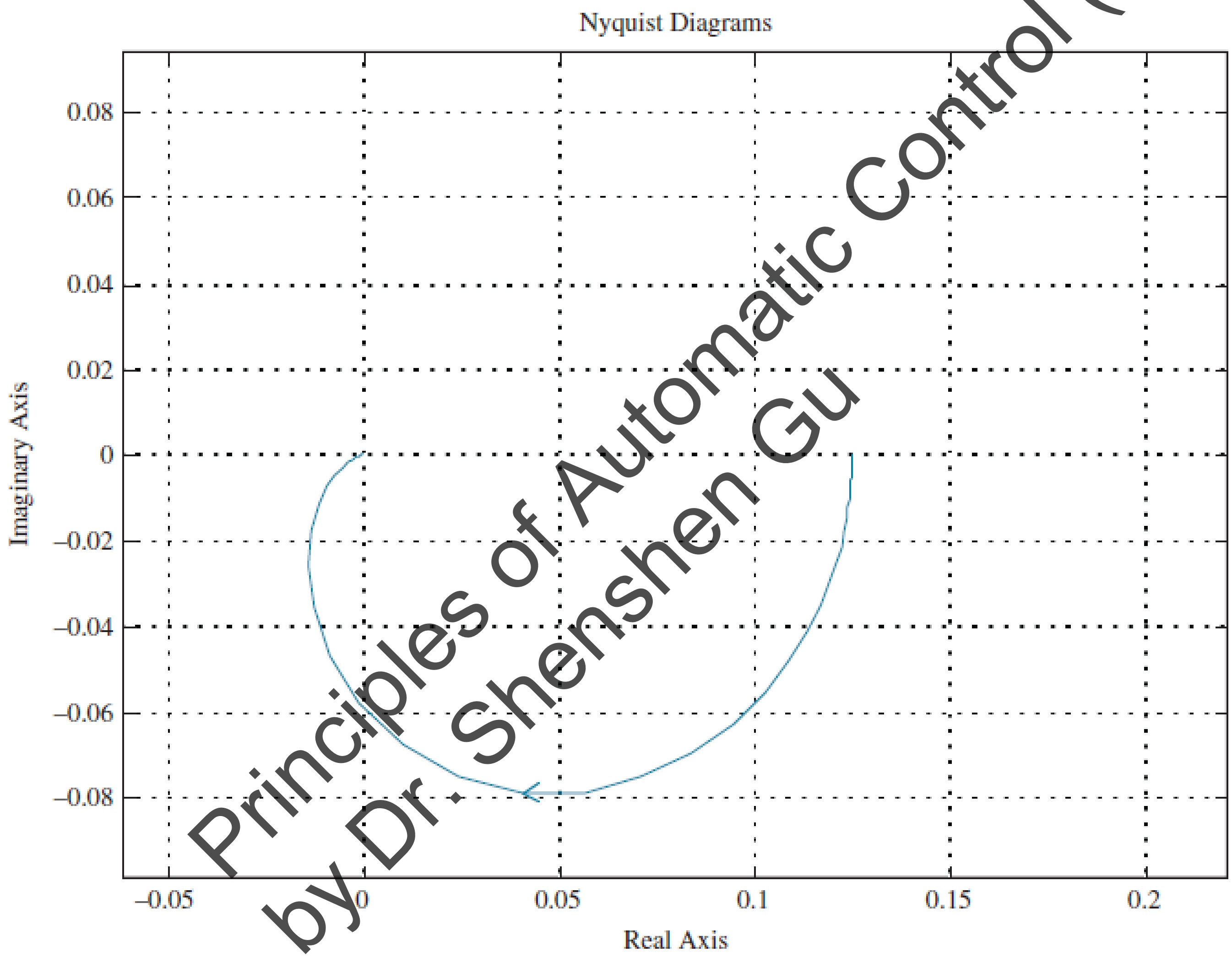
The complete solution is at www.wiley.com/college/nise.

a. $M(\omega) = \dfrac{1}{\sqrt{(8-\omega^2)^2 + (6\omega)^2}}$; for $\omega \le \sqrt{8}$: $\phi(\omega) = -\arctan\left(\dfrac{6\omega}{8-\omega^2}\right)$, for $\omega > \sqrt{8}$: $\phi(\omega) = -\left[\pi + \arctan\left(\dfrac{6\omega}{8-\omega^2}\right)\right]$

b.


Bode Diagrams
Phase (deg); Magnitude (dB)
0
–20
–40
–60
–80
–100
0
–50
–100
–150
–200
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)

c.



Nyquist Diagrams
0.08
0.06
0.04
0.02
0
–0.02
–0.04
–0.06
–0.08
Imaginary Axis
–0.05
0
0.05
0.1
0.15
0.2
Real Axis

2. Asymptotic Approximations: Bode Plots

- The log-magnitude and phase frequency response curves as functions of log ω are called **Bode plots** or **Bode diagrams**.
- Sketching Bode plots can be simplified because they can be approximated as a sequence of straight lines.
- Straight-line approximations simplify the evaluation of the magnitude and phase frequency response.

Magnitude response

$$G(s)=\frac{K(s+z_1)(s+z_2)\cdots(s+z_k)}{s^m(s+p_1)(s+p_2)\cdots(s+p_n)}$$

- The magnitude frequency response is the product of the magnitude frequency responses of each term, or

$$|G(s)|=\left.\frac{K|s+z_1||s+z_2|\cdots|s+z_k|}{|s^m||s+p_1||s+p_2|\cdots|s+p_n|}\right|_{s\to j\omega}$$

- If we know the magnitude response of each pole and zero term, we can find the total magnitude response.

Magnitude response

- Converting the magnitude response into dB, we obtain

$$20\log\left|G(s)\right| = 20\log K + 20\log\left|s+z_1\right| + 20\log\left|s+z_2\right| + \cdots + 20\log\left|s+z_k\right|$$
$$\left. - 20\log\left|s^m\right| - 20\log\left|s+p_1\right| - 20\log\left|s+p_2\right| - \cdots - 20\log\left|s+p_n\right|\right|_{s\to j\omega}$$

- Thus, if we knew the response of each term, the algebraic sum would yield the total response in dB.
- Further, if we could make an approximation of each term that would consist only of straight lines, graphical addition of terms would be greatly simplified.

Phase response

$$G(s)=\frac{K(s+z_1)(s+z_2)\cdots(s+z_k)}{s^m(s+p_1)(s+p_2)\cdots(s+p_n)}$$

- Let us look at the phase response. From the above equation, the phase frequency response is the sum of the phase frequency response curves of the zero terms minus the sum of the phase frequency response curves of the pole terms.

$$\angle G(s)=\angle K+\angle(s+z_1)+\angle(s+z_2)+\cdots+\angle(s+z_k)$$
$$-\angle(s^m)-\angle(s+p_1)-\angle(s+p_2)-\cdots-\angle(s+p_n)\Big|_{s\to j\omega}$$

- Again, since the phase response is the sum of individual terms, straight-line approximations to these individual responses simplify graphical addition.

Bode Plots for $G(s)=(s+a)$

Letting $s = j\omega$

$$G(j\omega)=(j\omega+a)=a\left(j\frac{\omega}{a}+1\right)$$

Low frequencies: ω<a

$$G(j\omega)\approx a$$

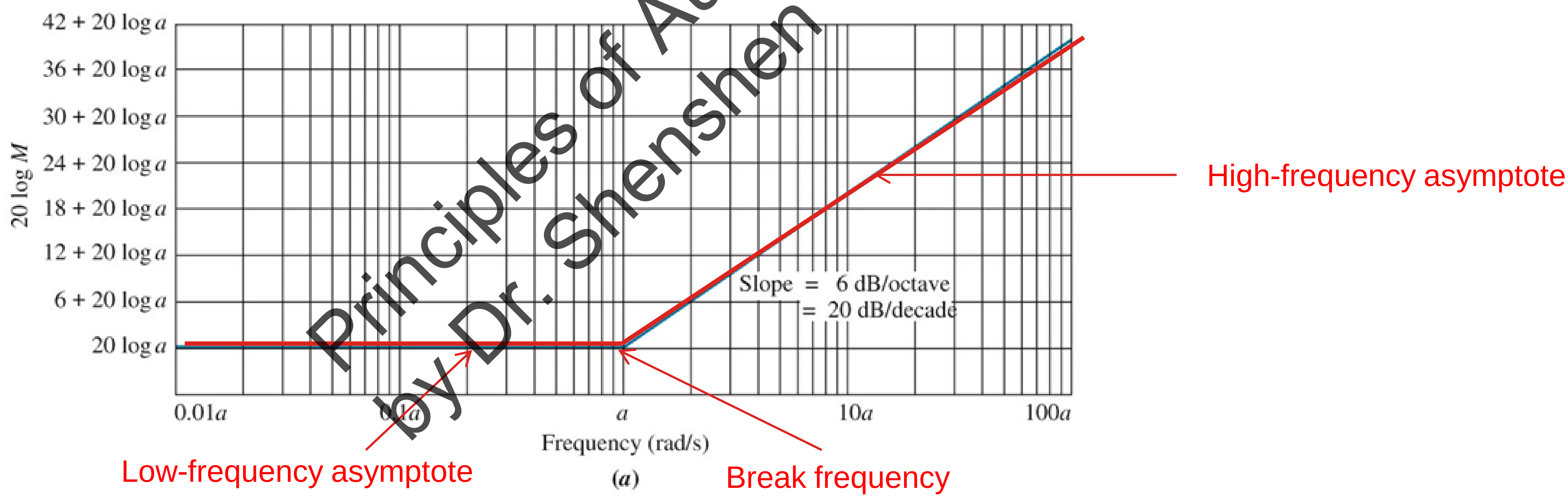
$$20\log M = 20\log a$$

High frequencies: ω>a

$$G(j\omega)\approx a\left(\frac{j\omega}{a}\right)=a\left(\frac{\omega}{a}\right)\angle 90^\circ$$

$$20\log M = 20\log a + 20\log\frac{\omega}{a} = 20\log\omega$$

plot dB, $20\log M$, against $\log\omega$ $\quad y = 20x$



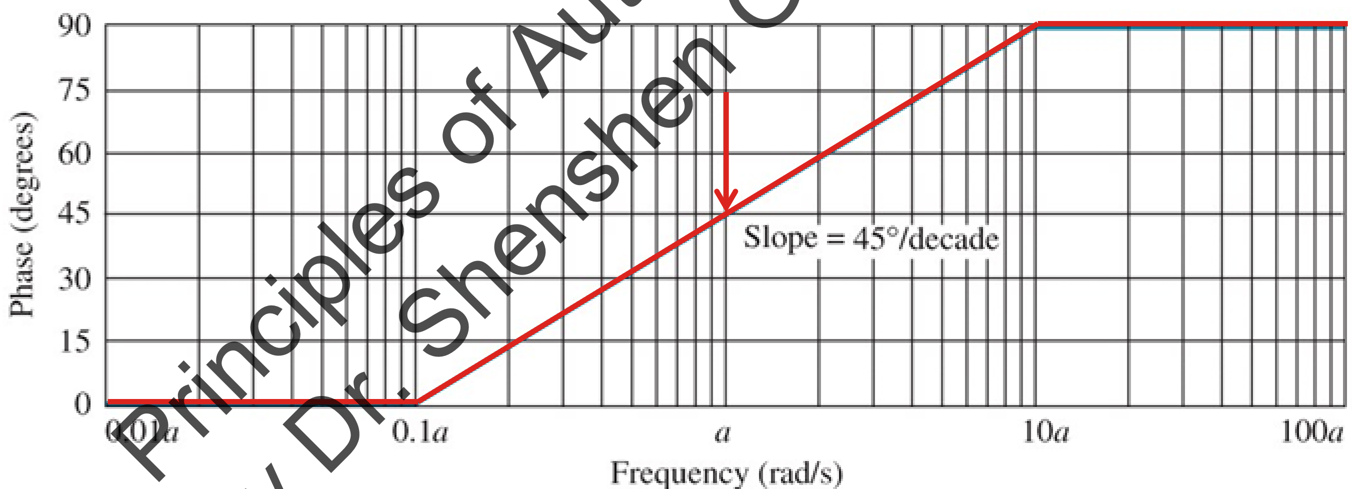
Phase response

$$G(j\omega) = (j\omega + a) = a\left(j\frac{\omega}{a} + 1\right)$$

$$G(j\omega)\big|_{\omega=a} = j\omega + a\big|_{\omega=a} = (ja + a) = \sqrt{2}a\angle 45^\circ$$

Low frequencies: $G(j\omega)\big|_{\omega \le 0.1a} \to \angle 0^\circ$

High frequencies: $G(j\omega)\big|_{\omega \ge 10a} \to \angle 90^\circ$



Normalized frequency response

- It is often convenient to normalize the magnitude so that the log-magnitude plot will be 0 dB at a break frequency.
  - When comparing different first- or second-order frequency response plots, each plot will have the same low-frequency asymptote after normalization;
  - When sketching the frequency response of each factor in the numerator and denominator will have the same low-frequency asymptote after normalization. This common low-frequency asymptote makes it easier to add components to obtain the Bode plot.

Normalize

First/second order term

$$G(s) = (s + a) = a\left(\frac{s}{a} + 1\right)$$

Constant term

Break frequency

Bode Plots for $G(s)=(s+a)$

$$G(s)=(s+a)=a\left(\frac{s}{a}+1\right)=\frac{1}{a}\times\left(\frac{s}{a}+1\right)$$

High frequencies: ω>a

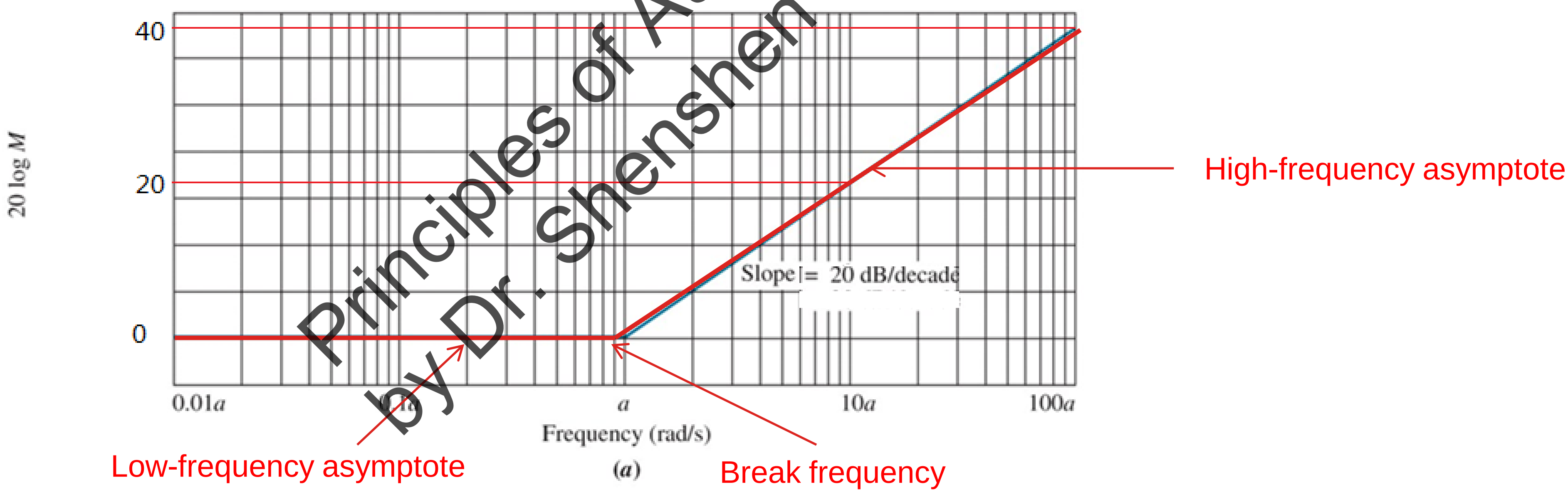
$$G(j\omega)\approx\left(\frac{j\omega}{a}\right)=\left(\frac{\omega}{a}\right)\angle 90^{\circ}$$

$$20\log M=20\log\frac{\omega}{a}$$

Low frequencies: ω<a

$$G(j\omega)\approx 1$$

$$20\log M=0$$



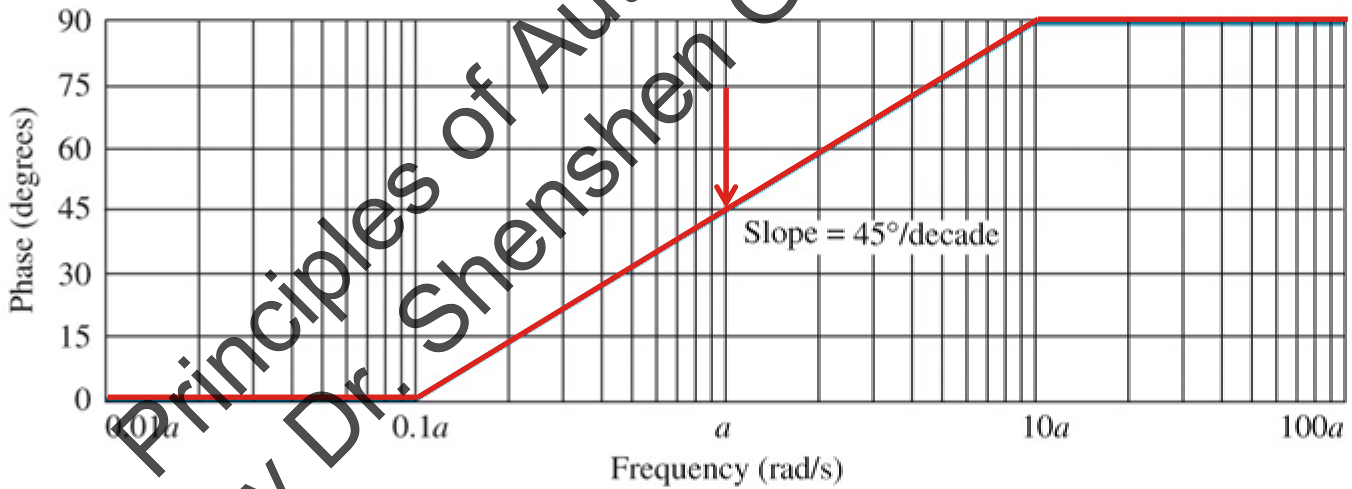
Phase response

$$G(j\omega) = j\frac{\omega}{a} + 1$$

$$G(j\omega)\Big|_{\omega=a} = j\frac{\omega}{a} + a\Big|_{\omega=a} = (j+1) = \sqrt{2}\angle 45^\circ$$

Low frequencies: $G(j\omega)\Big|_{\omega\le 0.1a} \to \angle 0^\circ$

High frequencies: $G(j\omega)\Big|_{\omega\ge 10a} \to \angle 90^\circ$



Bode Plots for $G(s)=1/(s+a)$

$$G(s)=\frac{1}{(s+a)}=\frac{1}{a\left(\frac{s}{a}+1\right)}=\frac{1}{a}\times\frac{1}{\left(\frac{s}{a}+1\right)}$$

Low frequencies

$$G(j\omega)\approx 1$$

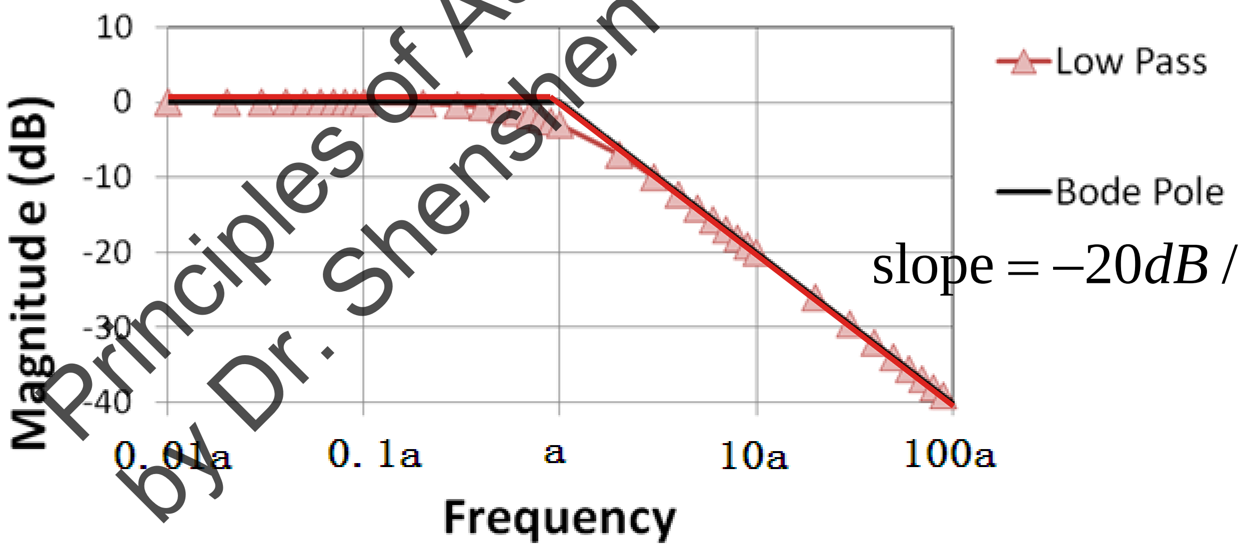
$$20\log M=20\log 1=0$$

High frequencies

$$G(j\omega)\approx\frac{1}{\left(\frac{j\omega}{a}\right)}=\frac{1}{\left(\frac{\omega}{a}\right)}\angle-90^\circ$$

$$20\log M=0-20\log\frac{\omega}{a}=-20\log\omega+20\log a$$

plot dB, $20\log M$, against $\log\omega$ $y=-20x$



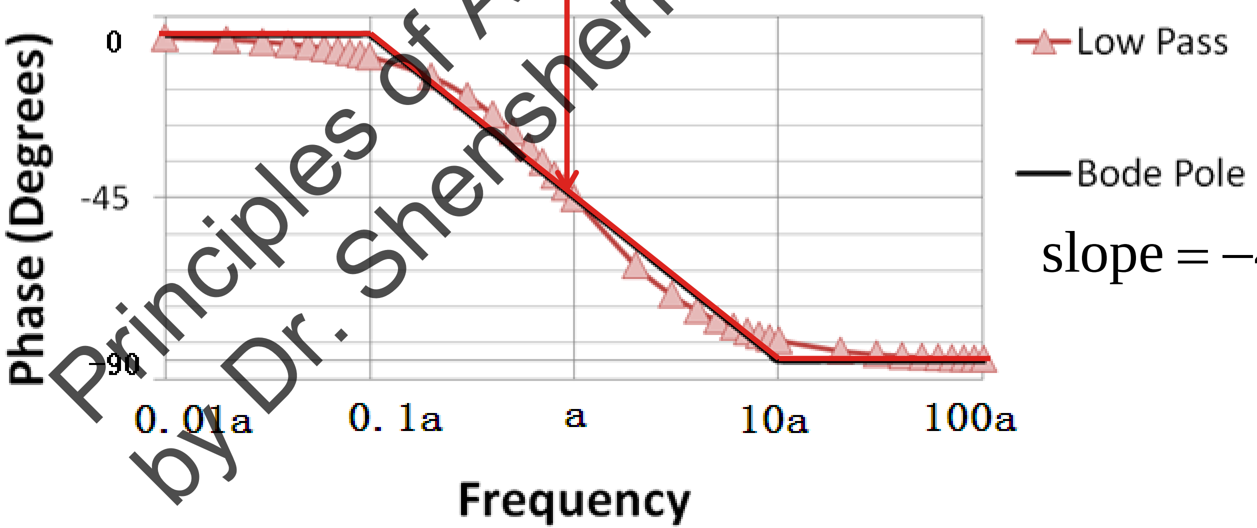
$$\text{slope}=-20dB/dec$$

Phase response

$$G(j\omega)\Big|_{\omega=a} = \frac{1}{j\omega+a}\Big|_{\omega=a} = \left(\frac{1}{ja+a}\right) = \frac{1}{\sqrt{2}a}\angle -45^\circ$$

Low frequencies: $G(j\omega)\big|_{\omega\le 0.1a} \to \angle 0^\circ$

High frequencies: $G(j\omega)\big|_{\omega\ge 10a} \to \angle -90^\circ$



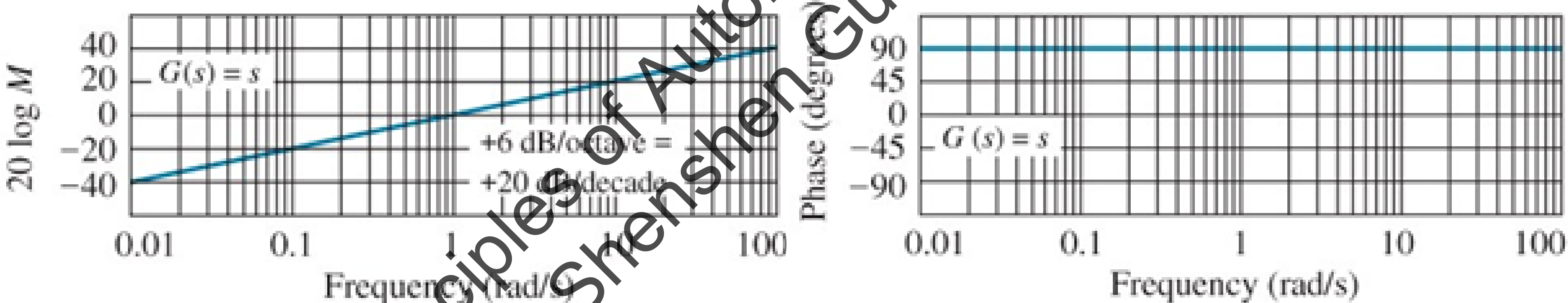
$\text{slope} = -45^\circ / dec$

Bode Plots for $G(s) = s$

Letting $s = j\omega$

$$G(j\omega) = j\omega = \omega\angle 90^\circ$$

$$20\log M = 20\log\omega$$

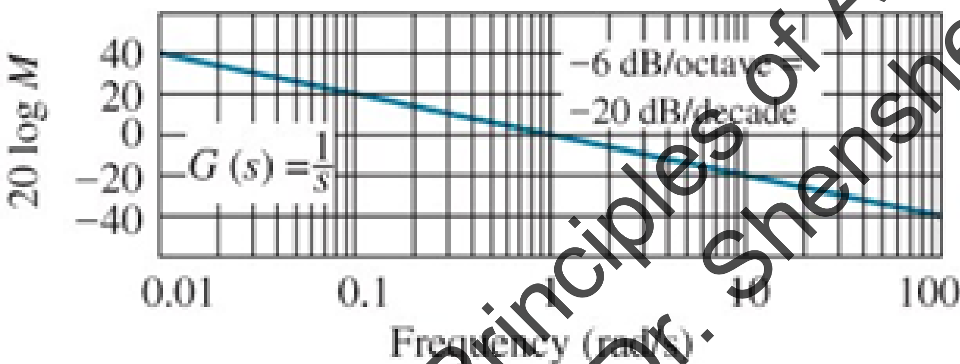


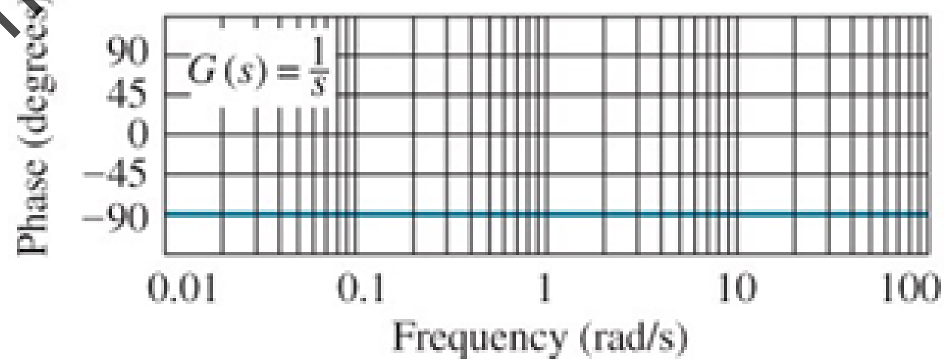
Bode Plots for $G(s) = 1/s$

Letting $s = j\omega$

$$G(j\omega) = \frac{1}{j\omega} = \frac{1}{\omega}\angle -90^\circ$$

$$20\log M = 20\log\frac{1}{\omega} = -20\log\omega$$





Example 10.2

Bode Plots for Ratio of First-Order Factors

FIGURE 10.10 Closed-loop unity feedback system

PROBLEM: Draw the Bode plots for the system shown in Figure 10.10, where $G(s) = K(s+3)/[s(s+1)(s+2)]$.

SOLUTION: We will make a Bode plot for the open-loop function $G(s) = K(s+3)/[s(s+1)(s+2)]$. The Bode plot is the sum of the Bode plots for each first-order term. Thus, it is convenient to use the normalized plot for each of these terms so that the low-frequency asymptote of each term, except the pole at the origin, is at 0 dB, making it easier to add the components of the Bode plot. We rewrite $G(s)$ showing each term normalized to a low-frequency gain of unity. Hence,

$$G(s) = \frac{\frac{3}{2}K\left(\frac{s}{3}+1\right)}{s(s+1)\left(\frac{s}{2}+1\right)} \tag{10.25}$$

Now determine that the break frequencies are at 1, 2, and 3. The magnitude plot should begin a decade below the lowest break frequency and extend a decade above the highest break frequency. Hence, we choose 0.1 radian to 100 radians, or three decades, as the extent of our plot.

$$G(s) = K\frac{(s+3)}{s(s+1)(s+2)} = \frac{\frac{3}{2}K\left(\frac{s}{3}+1\right)}{s(s+1)\left(\frac{s}{2}+1\right)}$$

$$\overset{K=1}{=} \frac{3}{2s} \times \left(\frac{s}{3}+1\right) \times \frac{1}{(s+1)} \times \frac{1}{\left(\frac{s}{2}+1\right)}$$

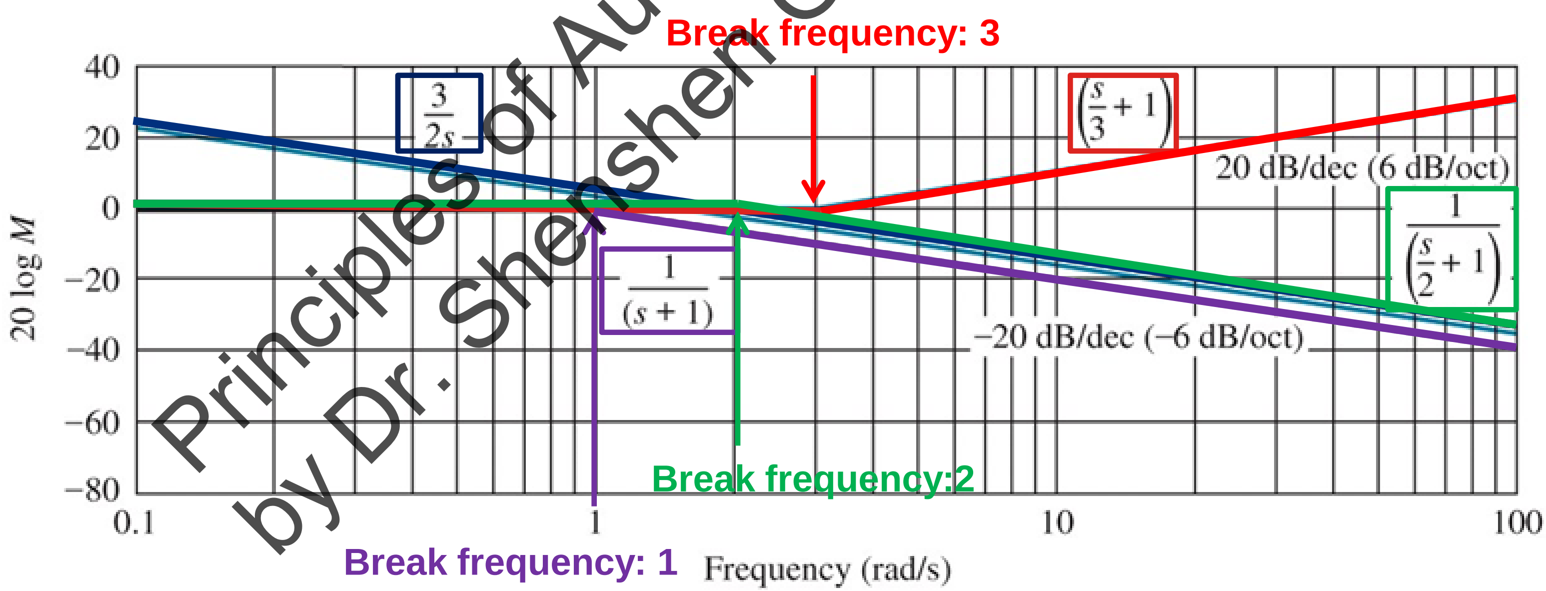
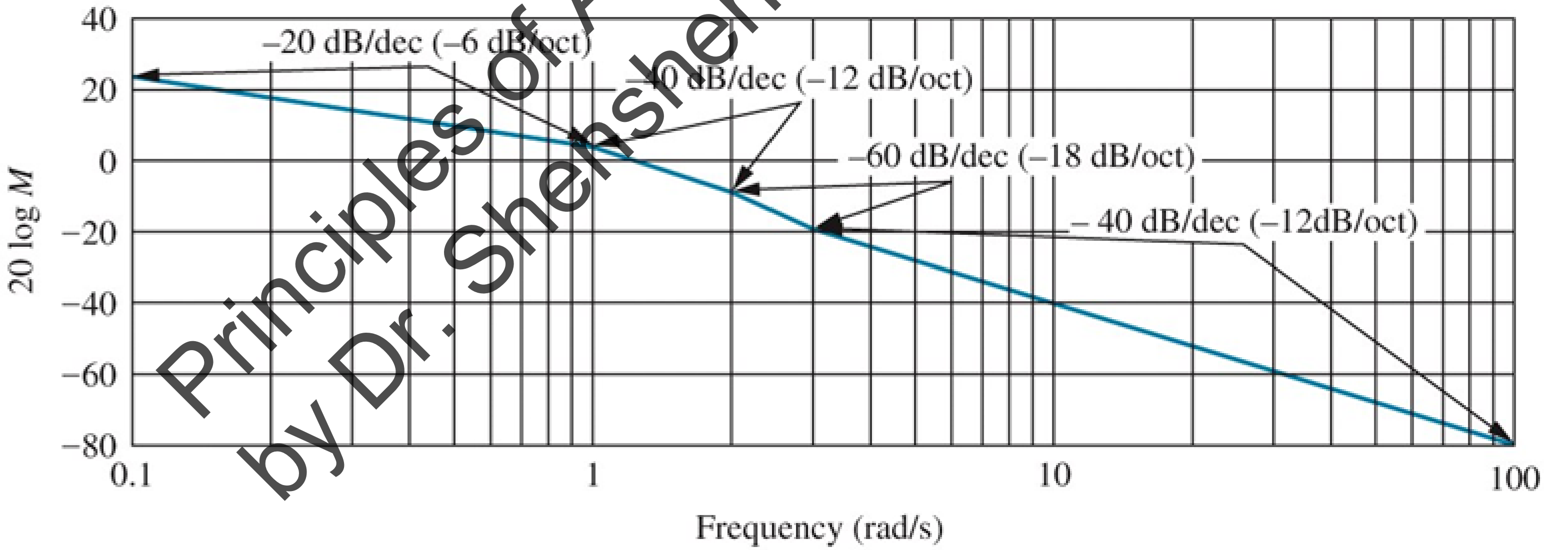


TABLE 10.2 Bode magnitude plot: slope contribution from each pole and zero in Example 10.2

	Frequency (rad/s)			
Description	**0.1 (Start: Pole at 0)**	**1 (Start: Pole at −1)**	**2 (Start: Pole at −2)**	**3 (Start: Zero at −3)**
Pole at 0	−20	−20	−20	−20
Pole at −1	0	−20	−20	−20
Pole at −2	0	0	−20	−20
Zero at −3	0	0	0	20
Total slope (dB/dec)	−20	−40	−60	−40

Table 10.2



$$G(s) = K\frac{(s+3)}{s(s+1)(s+2)} = \frac{\frac{3}{2}K\left(\frac{s}{3}+1\right)}{s(s+1)\left(\frac{s}{2}+1\right)}$$

$$\overset{K=1}{=} \frac{3}{2s} \times \left(\frac{s}{3}+1\right) \times \frac{1}{(s+1)} \times \frac{1}{\left(\frac{s}{2}+1\right)}$$

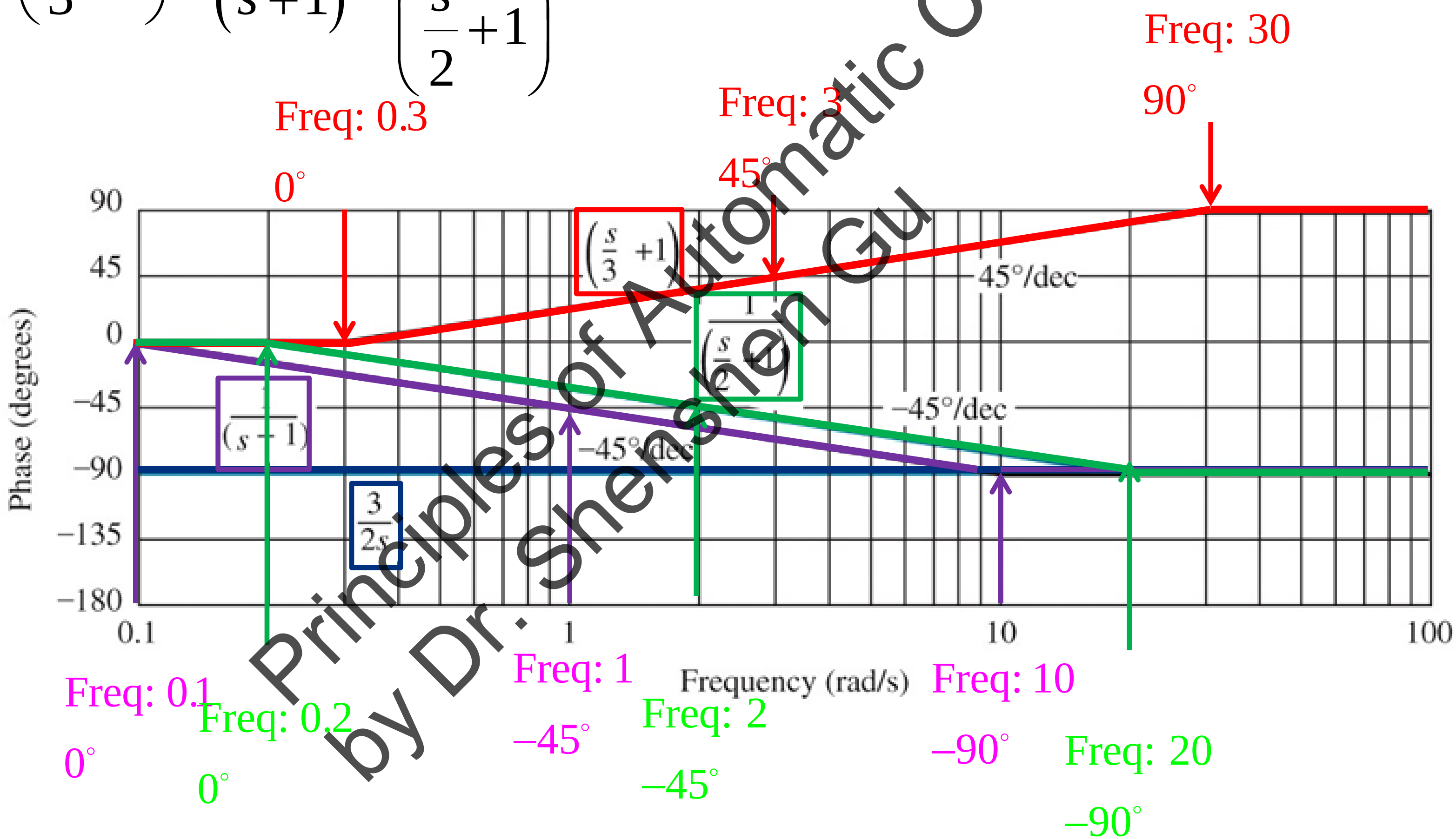
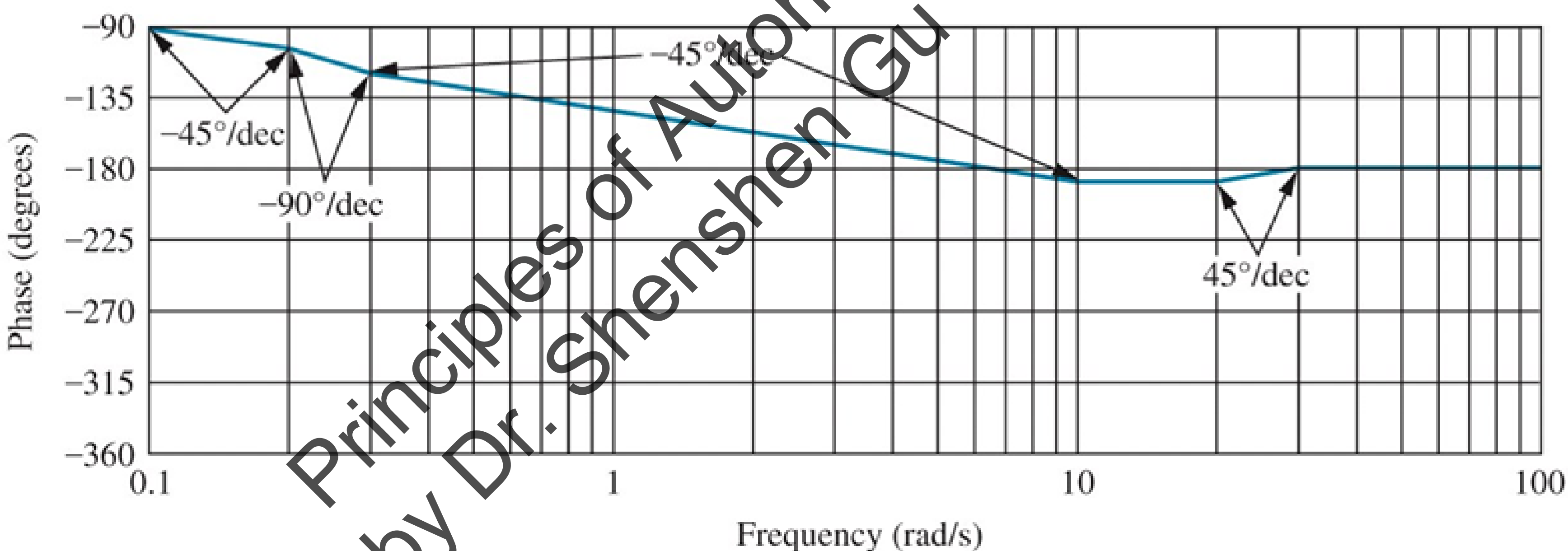


TABLE 10.3 Bode phase plot: slope contribution from each pole and zero in Example 10.2

	Frequency (rad/s)					
Description	**0.1 (Start: Pole at −1)**	**0.2 (Start: Pole at −2)**	**0.3 (Start: Pole at −3)**	**0 (End: Pole at −1)**	**20 (End: Pole at −2)**	**30 (End: Zero at −3)**
Pole at −1	−45	−45	−45	0		
Pole at −2		−45	−45	−45	0	
Zero at −3			45	45	45	0
Total slope (deg/dec)	−45	−90	−45	0	45	0

Table 10.3



Skill-Assessment Exercise 10.2

PROBLEM: Draw the Bode log-magnitude and phase plots for the system shown in Figure 10.10, where

$$G(s) = \frac{(s+20)}{(s+1)(s+7)(s+50)}$$

WileyPLUS

WPCS

Control Solutions

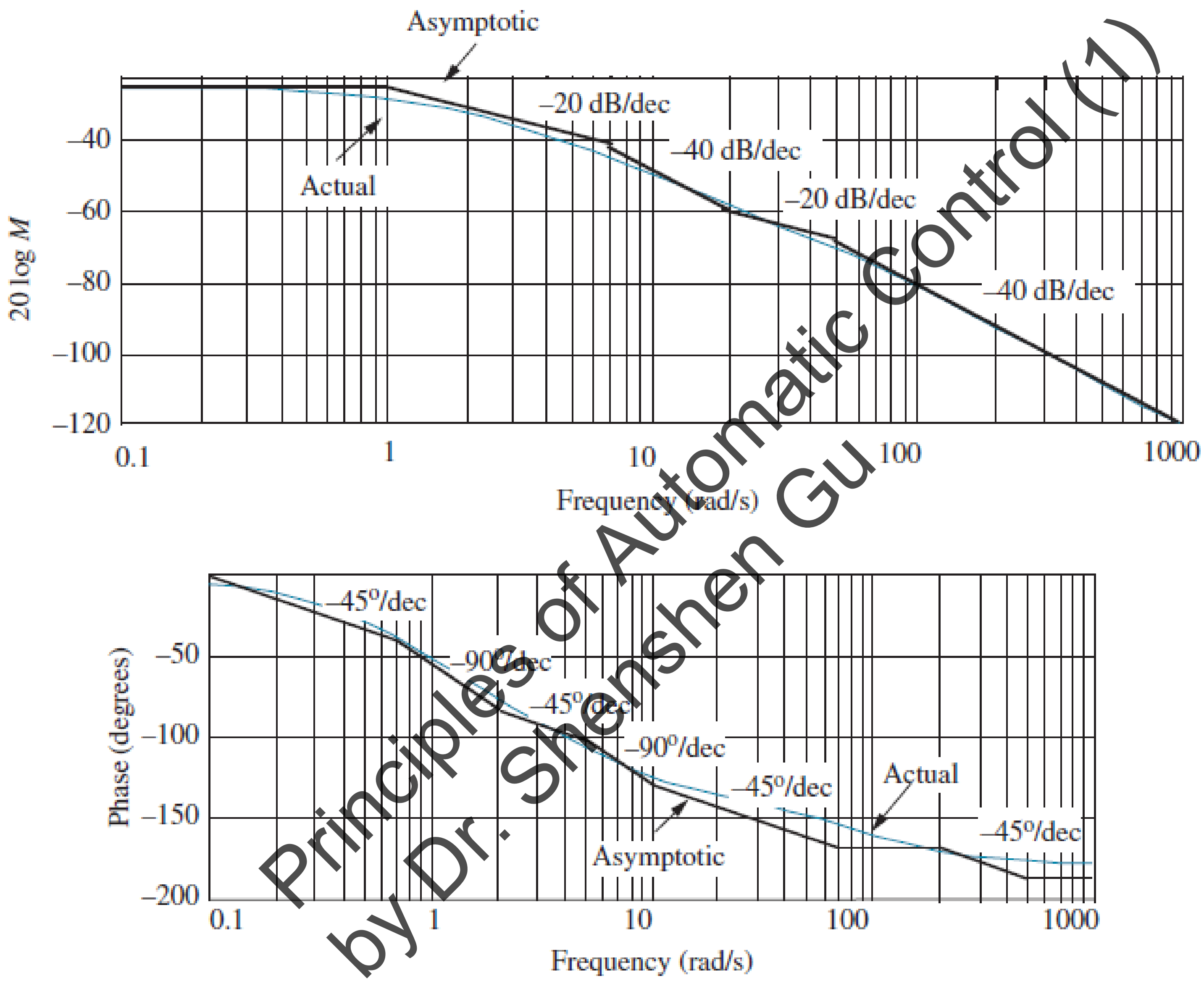
ANSWER: The complete solution is at www.wiley.com/college/nise.

TryIt 10.1

Use MATLAB, the Control System Toolbox, and the following statements to obtain the Bode plots for the system of Skill-Assessment Exercise 10.2

```
G=zpk([-20],[-1,-7,...
 -50],1)
bode(G);grid on
```

After the Bode plots appear, click on the curve and drag to read the coordinates.



Asymptotic
Actual
–20 dB/dec
–40 dB/dec
–20 dB/dec
–40 dB/dec
20 log M
–40
–60
–80
–100
–120
0.1
1
10
100
1000
Frequency (rad/s)
–45°/dec
–90°/dec
–45°/dec
–90°/dec
–45°/dec
Actual
–45°/dec
Asymptotic
Phase (degrees)
–50
–100
–150
–200
0.1
1
10
100
1000
Frequency (rad/s)

Bode Plots for $G(s)=s^2+2\zeta\omega_n s+\omega_n^2$

$$G(s)=s^2+2\zeta\omega_n s+\omega_n^2=\omega_n^2\left(\frac{s^2}{\omega_n^2}+2\zeta\frac{s}{\omega_n}+1\right)$$

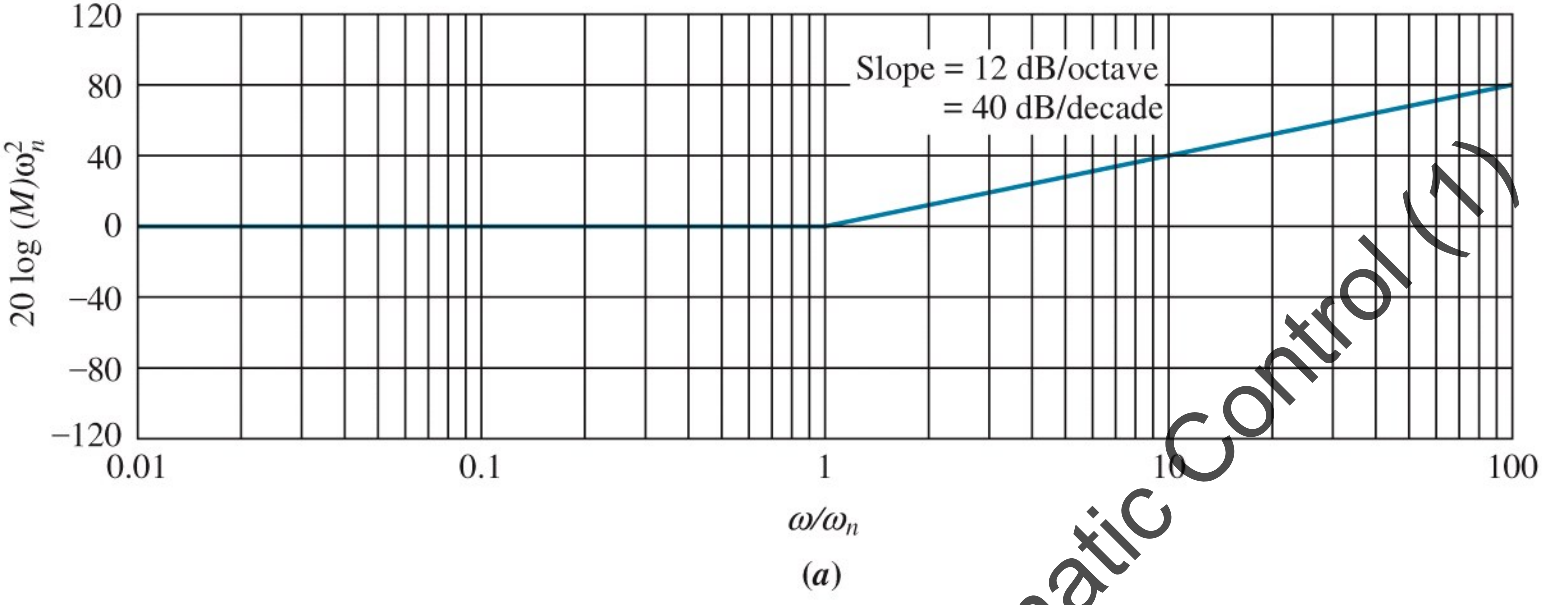
low frequency: $\left(\frac{s^2}{\omega_n^2}+2\zeta\frac{s}{\omega_n}+1\right)\approx 1$ high frequency: $\left(\frac{s^2}{\omega_n^2}+2\zeta\frac{s}{\omega_n}+1\right)\approx \frac{s^2}{\omega_n^2}$

$$1\Big|_{s=j\omega}=1\angle 0^\circ$$

$$20\log M=20\log 1=0$$

$$\left.\frac{s^2}{\omega_n^2}\right|_{s=j\omega}=-\frac{\omega^2}{\omega_n^2}=\frac{\omega^2}{\omega_n^2}\angle 180^\circ$$

$$20\log M=20\log\frac{\omega^2}{\omega_n^2}=40\log\omega-40\log\omega_n$$


120
80
40
0
−40
−80
−120
$20 \log (M)\omega_n^2$
Slope = 12 dB/octave
= 40 dB/decade
0.01
0.1
1
10
100
ω/ω_n
(a)

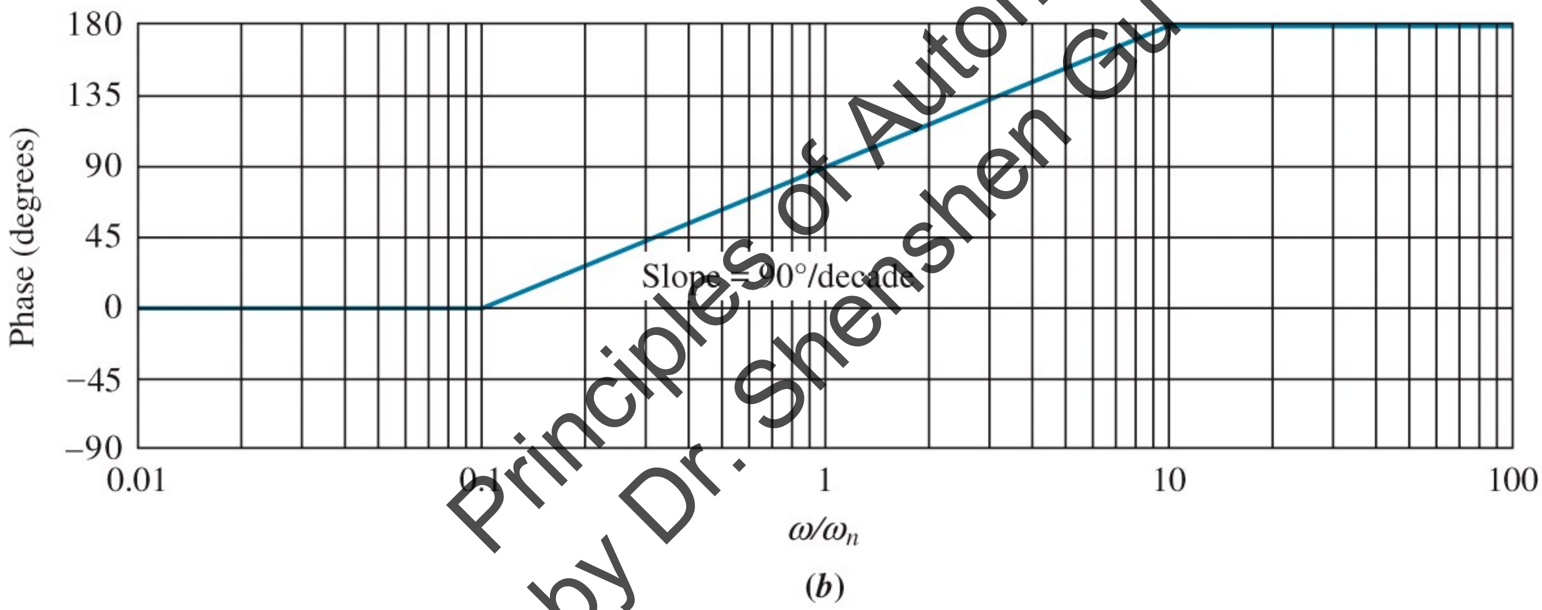

180
135
90
45
0
−45
−90
Phase (degrees)
Slope = 90°/decade
0.01
0.1
1
10
100
ω/ω_n
(b)

Figure 10.13

Bode Plots for $G(s)=1/\left(s^2+2\zeta\omega_n s+\omega_n^2\right)$

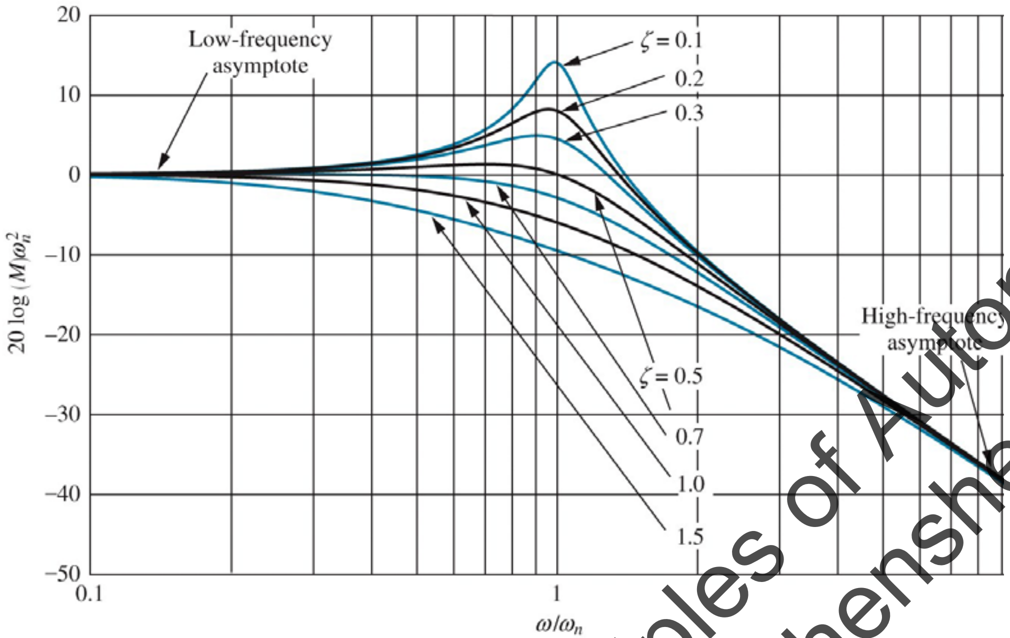


Figure 10.16

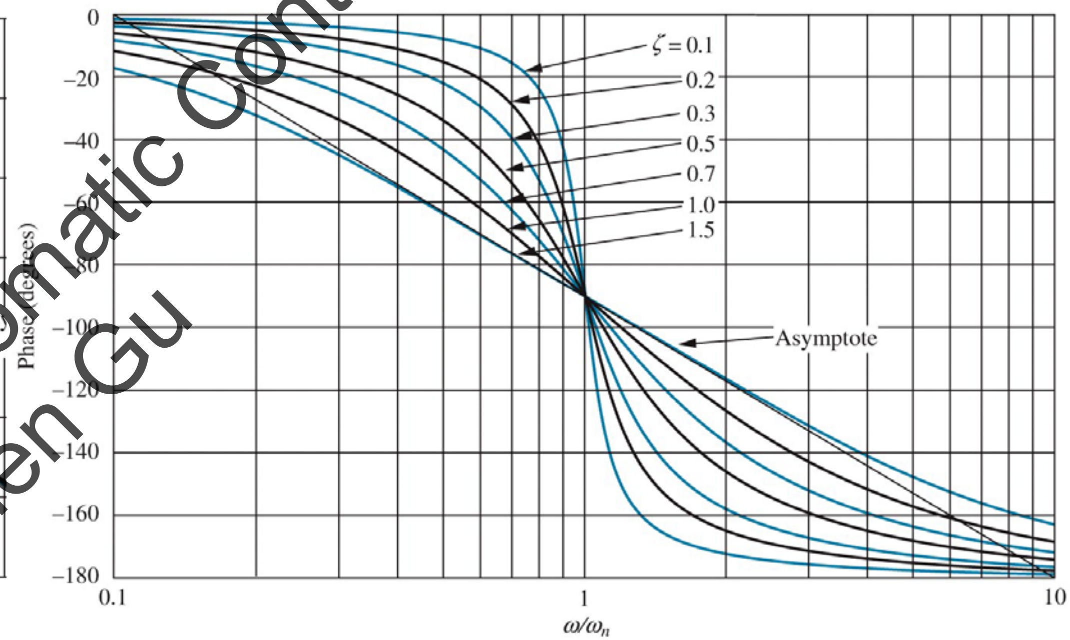


Figure 10.17

3. Introduction to the Nyquist Criterion

- The **Nyquist criterion** relates the stability of a closed-loop system to the open-loop frequency response and open-loop pole location.
- Thus, knowledge of the open-loop system's frequency response yields information about the stability of the closed-loop system.
- This concept is similar to the **root locus**, where we began with information about the open-loop system, its poles and zeros, and developed transient and stability information about the closed-loop system.

Derivation of the Nyquist Criterion

- The Nyquist criterion can tell us how many closed loop poles are in the right half-plane.
- Let us establish four important concepts that will be used during the derivation:
  - the relationship between the poles of 1+G(s)H(s) and the poles of G(s)H(s);
  - the relationship between the zeros of 1+G(s)H(s) and the poles of the closed-loop transfer function, T(s);
  - the concept of mapping points;
  - the concept of mapping contours.

Letting $G(s) = \frac{N_G}{D_G}, \ H(s) = \frac{N_H}{D_H}$

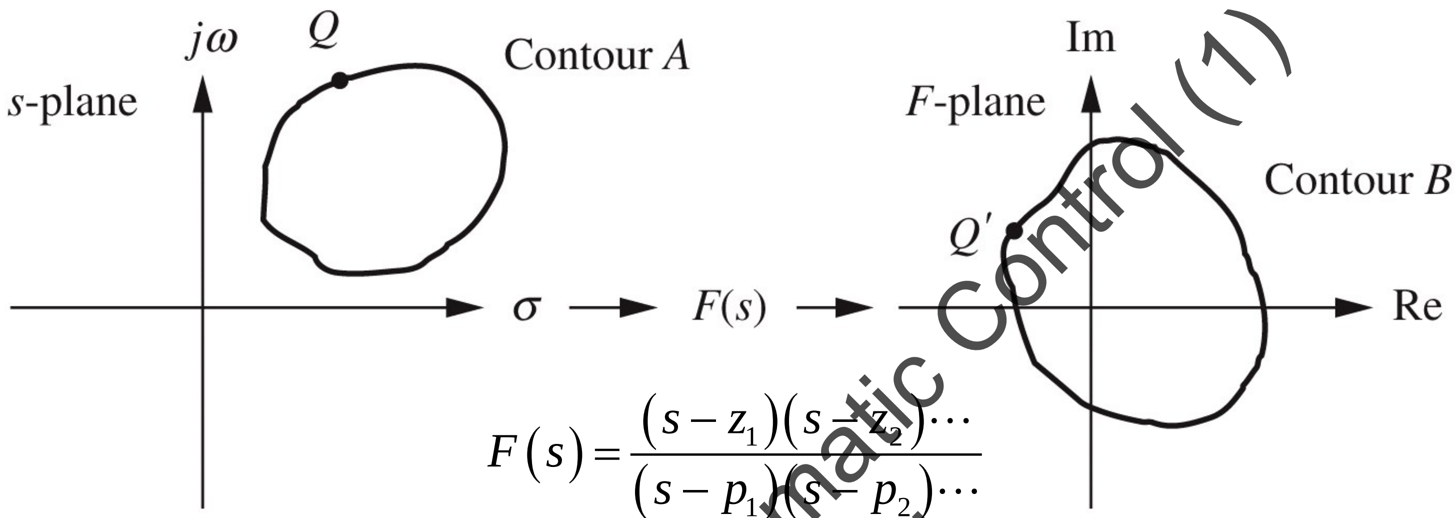
We find $G(s)H(s) = \frac{N_G N_H}{D_G D_H}$

$$1 + G(s)H(s) = 1 + \frac{N_G N_H}{D_G D_H} = \frac{D_G D_H + N_G N_H}{D_G D_H}$$

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{N_G D_H}{D_G D_H + N_G N_H}$$

- We conclude that:
- (1) the poles of *1+G(s)H(s)* are the same as the poles of *G(s)H(s)*, the open-loop system, and
- (2) the zeros of *1+G(s)H(s)* are the same as the poles of *T(s)*, the closed-loop system.

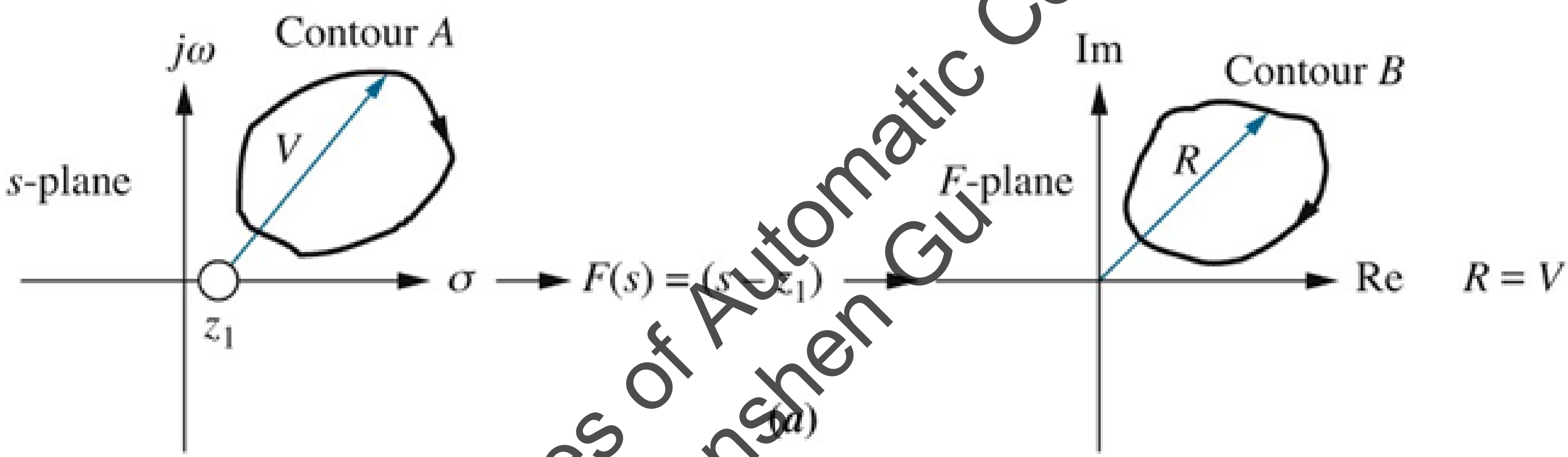
- Let us define the term ***mapping***. If we take a complex number on the s-plane and substitute it into a function, F(s), another complex number results. This process is called *mapping*.
- For example, substituting $s=4+j3$ into the function (s^2+2s+1) yields $16+j30$. We say that $4+j3$ maps into $16+j30$ through the function (s^2+2s+1) .
- Finally, we discuss the concept of mapping contours.



$$F(s)=\frac{(s-z_1)(s-z_2)\cdots}{(s-p_1)(s-p_2)\cdots}$$

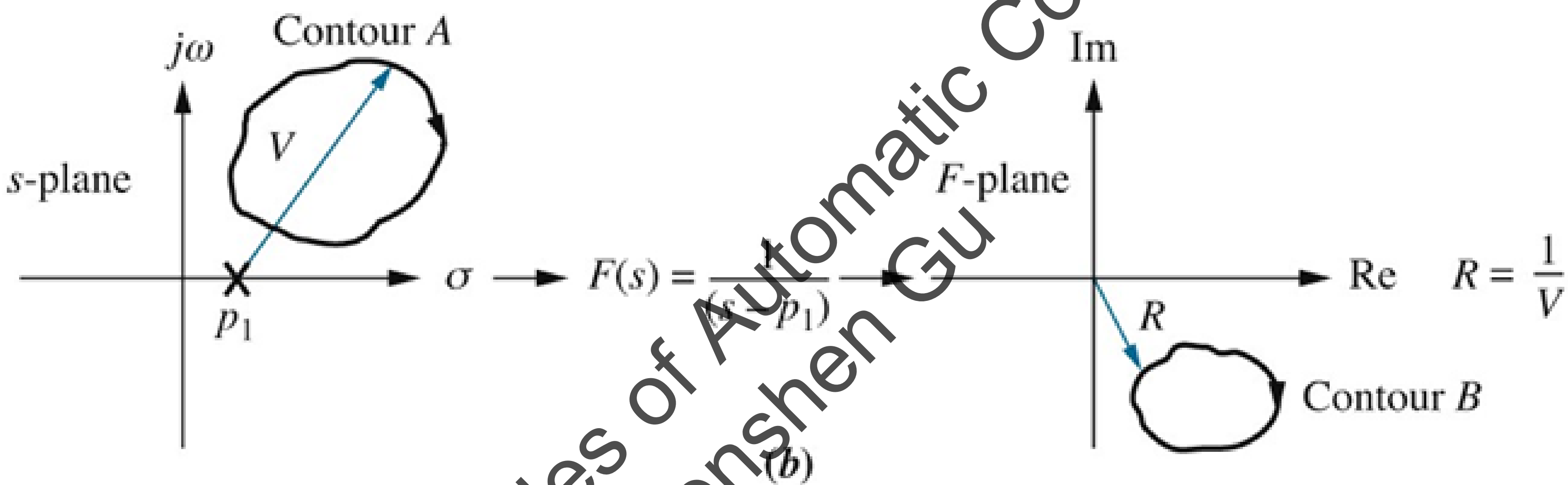
Figure 10.21

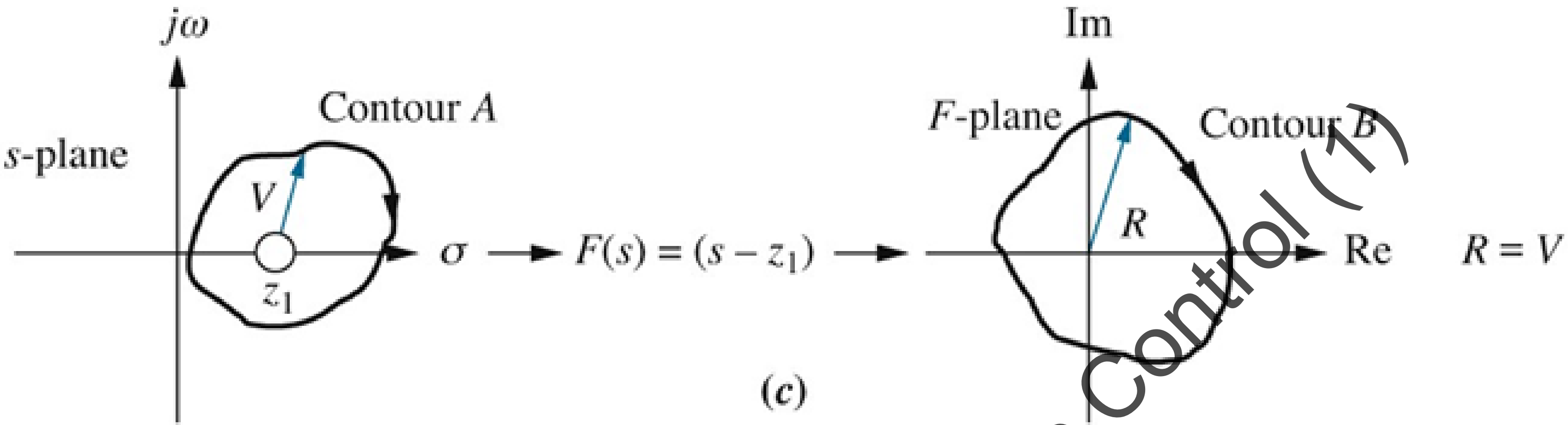
- The vector approach to performing the calculation, can be used as an alternative.
- Some examples of contour mapping are shown for some simple F(s).
- The mapping of each point is defined by complex arithmetic, where the resulting complex number, *R*, is evaluated from the complex numbers represented by *V.*

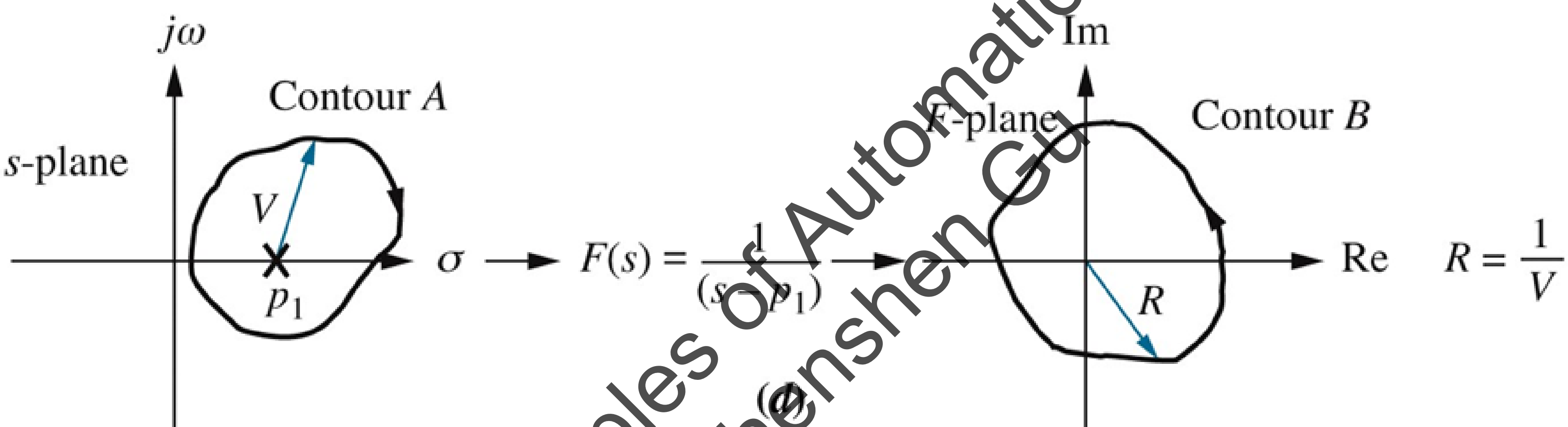


You should verify that if we assume a clockwise direction for mapping the points on contour A, then contour B maps in a clockwise direction if F(s) has just zeros or …

has just poles that are not encircled by the contour.

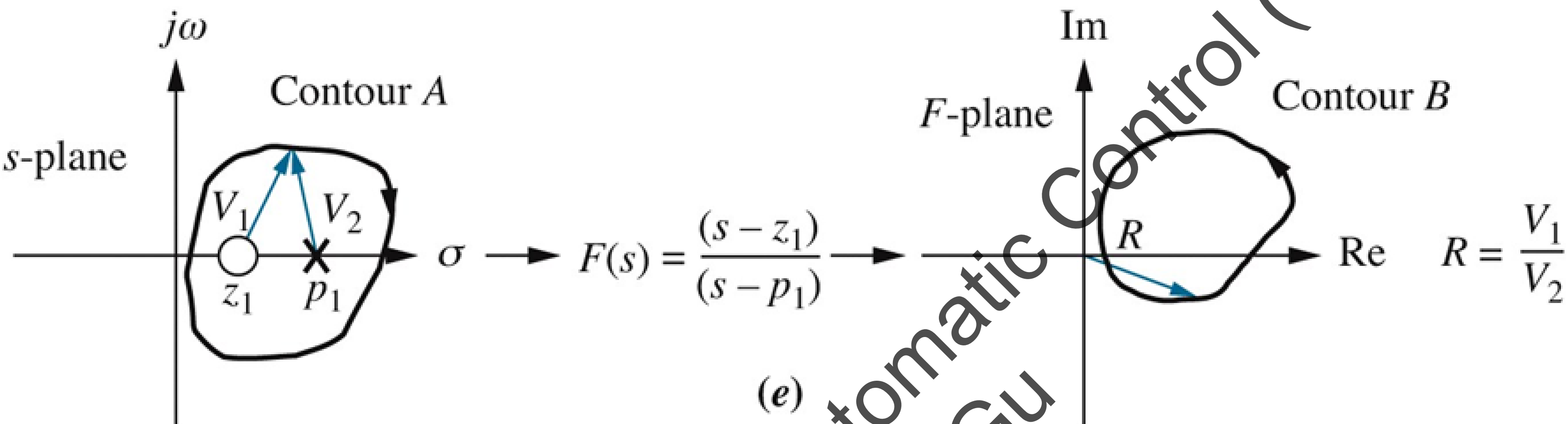






The contour B maps in a counterclockwise direction if F(s) has just poles that are encircled by the contour.

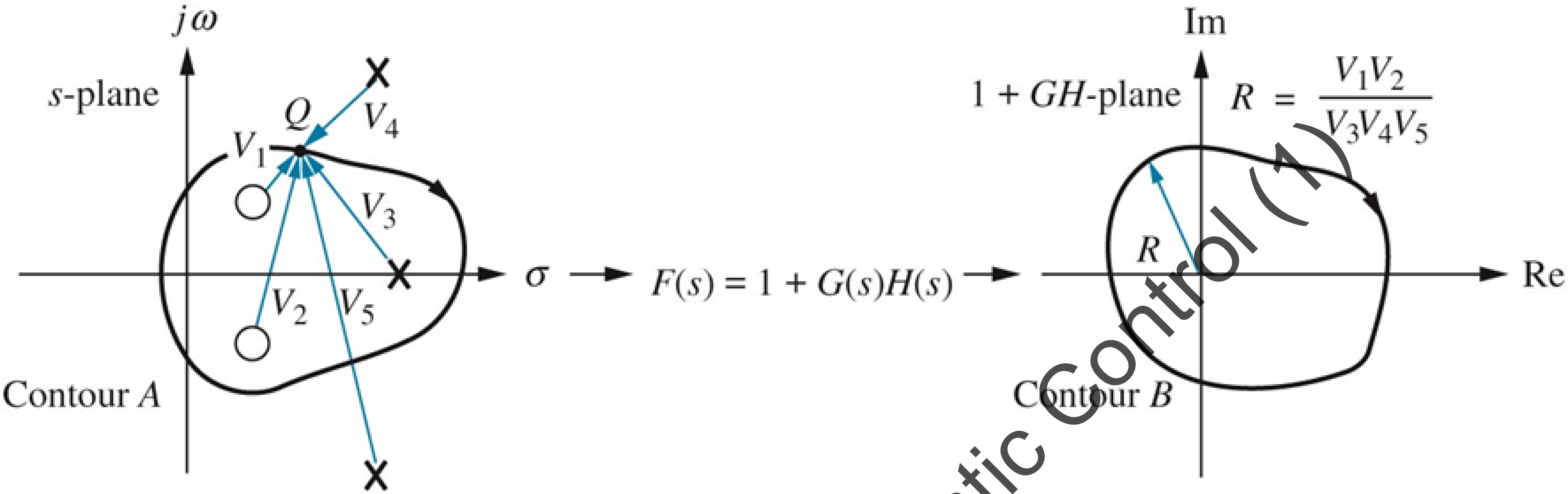
Also, you should verify that if the pole or zero of F(s) is enclosed by contour A, the mapping encircles the origin.



In the last case, the pole and zero rotation cancel, and the mapping does not encircle the origin.

Nyquist criterion

- We show that a unique relationship exists between the number of poles of F(s) contained inside contour A, the number of zeros of F(s) contained inside contour A, and the number of ***counterclockwise*** encirclements of the origin for the mapping of contour B.
- We then show how this interrelationship can be used to determine the stability of closed-loop systems.
- This method of determining stability is called the Nyquist criterion.



- As we move around contour A in a clockwise direction, each vector that lies inside contour A will appear to undergo a complete rotation, or a change in angle of 360° .
- On the other hand, each vector drawn from the poles and zeros of 1+G(s)H(s) that exist outside contour A will appear to oscillate and return to its previous position, undergoing a net angular change of 0°.

- Each pole or zero factor of 1+G(s)H(s) whose vector undergoes a complete rotation around contour A must yield a change of 360° in the resultant, R, or a complete rotation of the mapping of contour B.
- If we move in a clockwise direction along contour A, each zero inside contour A yields a rotation in the clockwise direction, while each pole inside contour A yields a rotation in the counterclockwise direction since poles are in the denominator

$$N = P - Z$$

- N equals the number of counterclockwise rotations of contour B about the origin;
- P equals the number of poles of 1+G(s)H(s) inside contour A;
  - they are also the poles of G(s)H(s) and are known.
- Z equals the number of zeros of 1+G(s)H(s) inside contour A.
  - they are also the poles of the closed-loop system and are **NOT** known.

$$Z = P - N$$

- Tells us that the number of closed-loop poles inside the contour (which is the same as the zeros of 1+G(s)H(s) inside the contour) equals the number of open-loop poles of G(s)H(s) inside the contour minus the number of counterclockwise rotations of the mapping about the origin.

Extend the contour

- If we extend the contour to include the entire right half-plane, we can count the number of right–half-plane, closed-loop poles inside contour A and determine a system's stability.
- Since we can count the number of open loop poles, P, inside the contour, which are the same as the right–half-plane poles of G(s)H(s), the only problem remaining is how to obtain the mapping and find N.

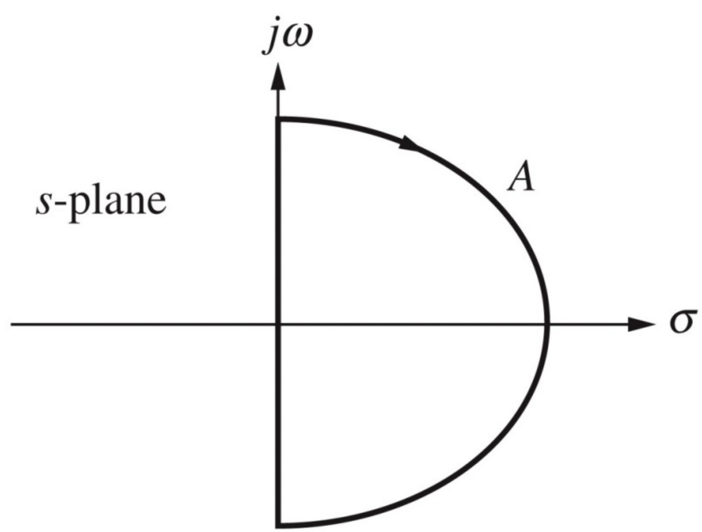
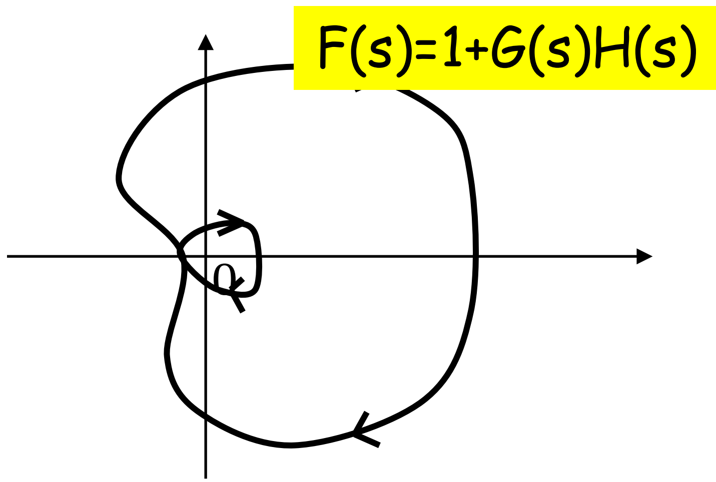


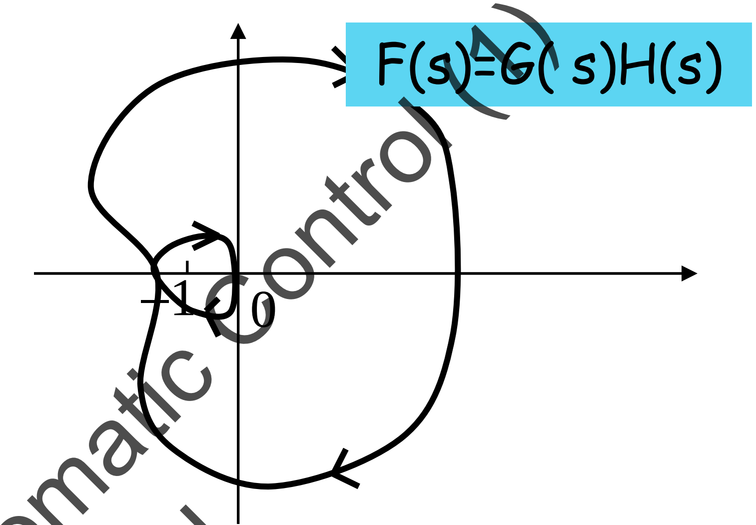
Figure 10.24

1+G(s)H(s) vs. G(s)H(s)

- Since all of the poles and zeros of G(s)H(s) are known, what if we map through G(s)H(s) instead of 1+G(s)H(s)? The resulting contour is the same as a mapping through 1+G(s)H(s), except that it is translated one unit to the left; thus, we count rotations about **-1** instead of rotations about the origin. Hence, the final statement of the Nyquist stability criterion is as follows:

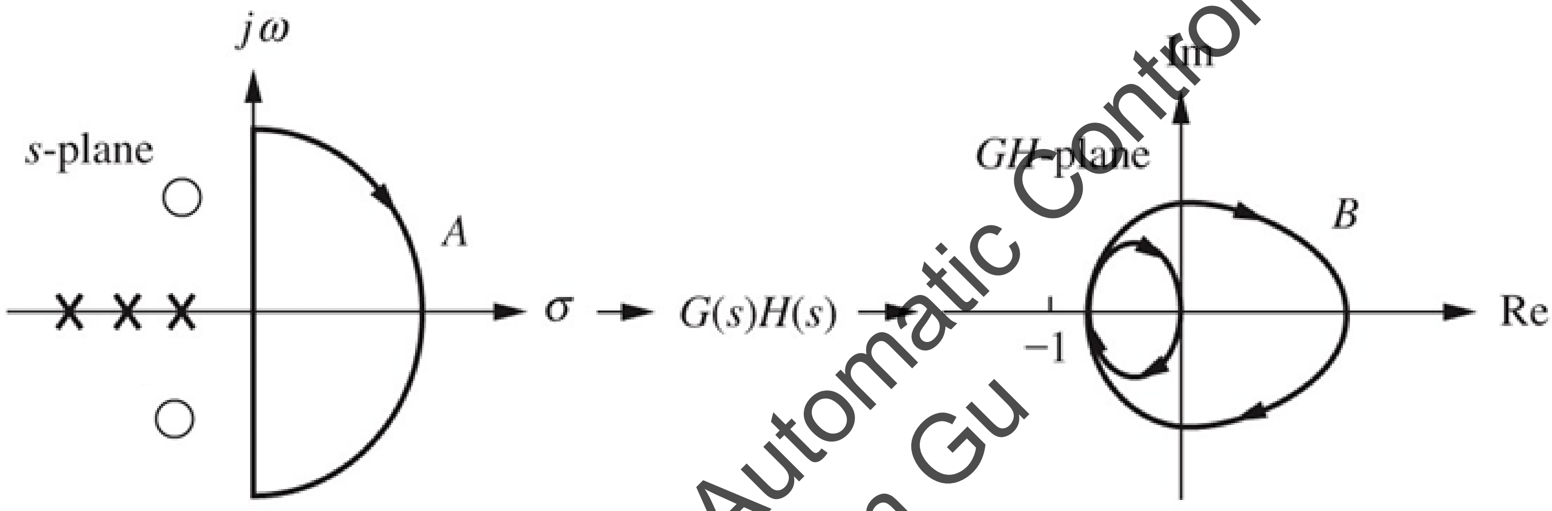
- If a contour, A, that encircles the entire right half-plane is mapped through G(s)H(s), then the number of closed-loop poles, Z, in the right half-plane equals the number of open-loop poles, P, that are in the right half-plane minus the number of counterclockwise revolutions, N, around **-1** of the mapping; that is, Z=P-N. The mapping is called the Nyquist diagram, or Nyquist plot, of G(s)H(s).





- We can now see why this method is classified as a frequency response technique. Around contour A, the mapping of the points on the *jω*-axis through the function *G(s)H(s)* is the same as substituting *s= jω* into *G(s)H(s)* to form the frequency response function *G(jω)H(jω)*.We are thus finding the frequency response of *G(s)H(s)* over that part of contour A on the positive *jω*-axis. In other words, part of the Nyquist diagram is the polar plot of the frequency response of *G(s)H(s)*.

Applying the Nyquist Criterion to Determine Stability



(a)

○ = zeros of $1 + G(s)H(s)$
= poles of closed-loop system
Location not known

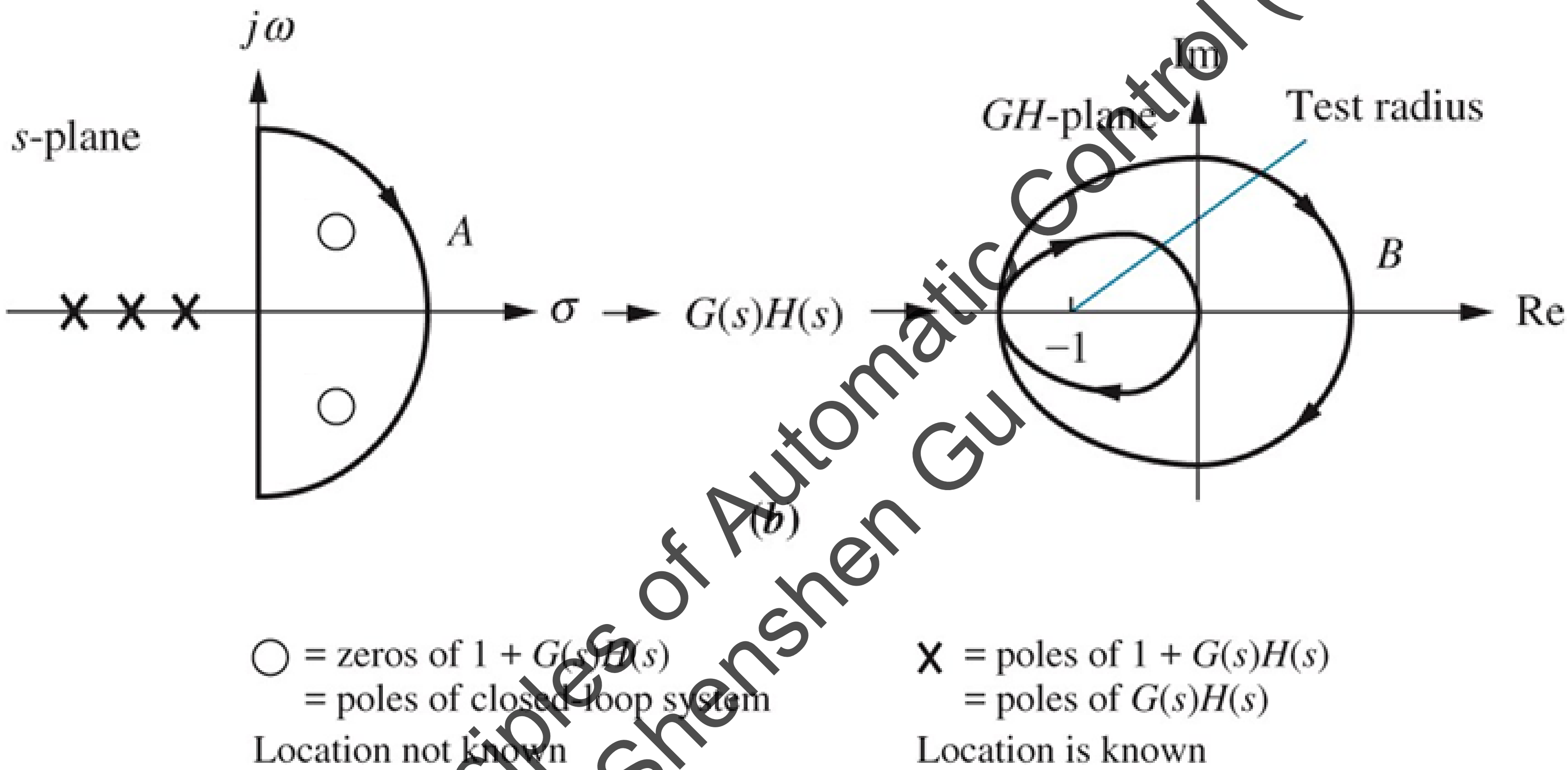
X = poles of $1 + G(s)H(s)$
= poles of $G(s)H(s)$
Location is known

The contour maps through G(s)H(s) into a Nyquist diagram does not encircle ***-1. Hence N=0***

Contour A does not encircle any poles of G(s)H(s)***. Hence P=0***

Z=P-N=0. This system has no right–half-plane poles and is stable.

Applying the Nyquist Criterion to Determine Stability



The contour maps through G(s)H(s) into a Nyquist diagram generate two clockwise encirclements of ***-1. Hence N=-2***

Contour A does not encircle any poles of G(s)H(s)***. Hence P=0***

Z=P-N=2. This system has 2 right–half-plane poles and is unstable.

4. Sketching the Nyquist Diagram

- The contour that encloses the right half-plane can be mapped through the function G(s)H(s) by substituting points along the contour into G(s)H(s).
- The points along the positive extension of the imaginary axis yield the polar frequency response of G(s)H(s).

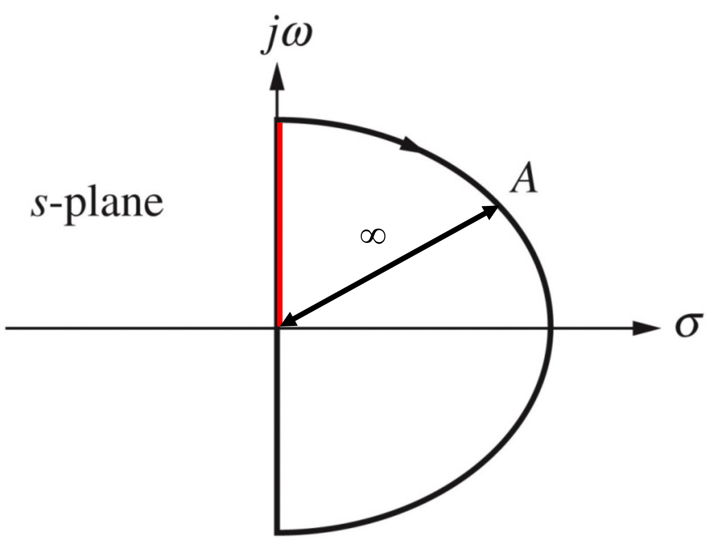
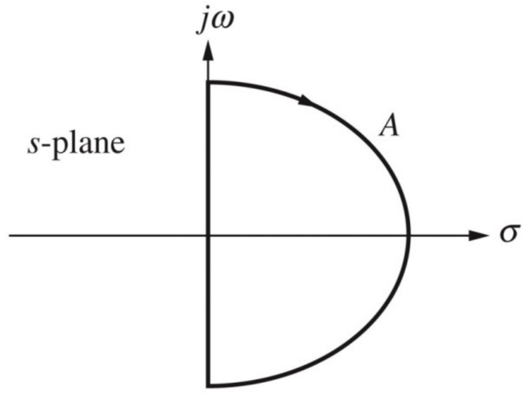


Figure 10.24

- Approximations can be made to G(s)H(s) for points around the infinite semicircle by assuming that the vectors originate at the origin.
  - Their length is infinite, and their angles are easily evaluated.
- However, most of the time a simple sketch of the Nyquist diagram is all that is needed.
- A sketch can be obtained ***rapidly*** by looking at the vectors of G(s)H(s) and their motion along the contour.



Example 10.4

Sketching a Nyquist Diagram

PROBLEM: Speed controls find wide application throughout industry and the home. Figure 10.26(*a*) shows one application: output frequency control of electrical

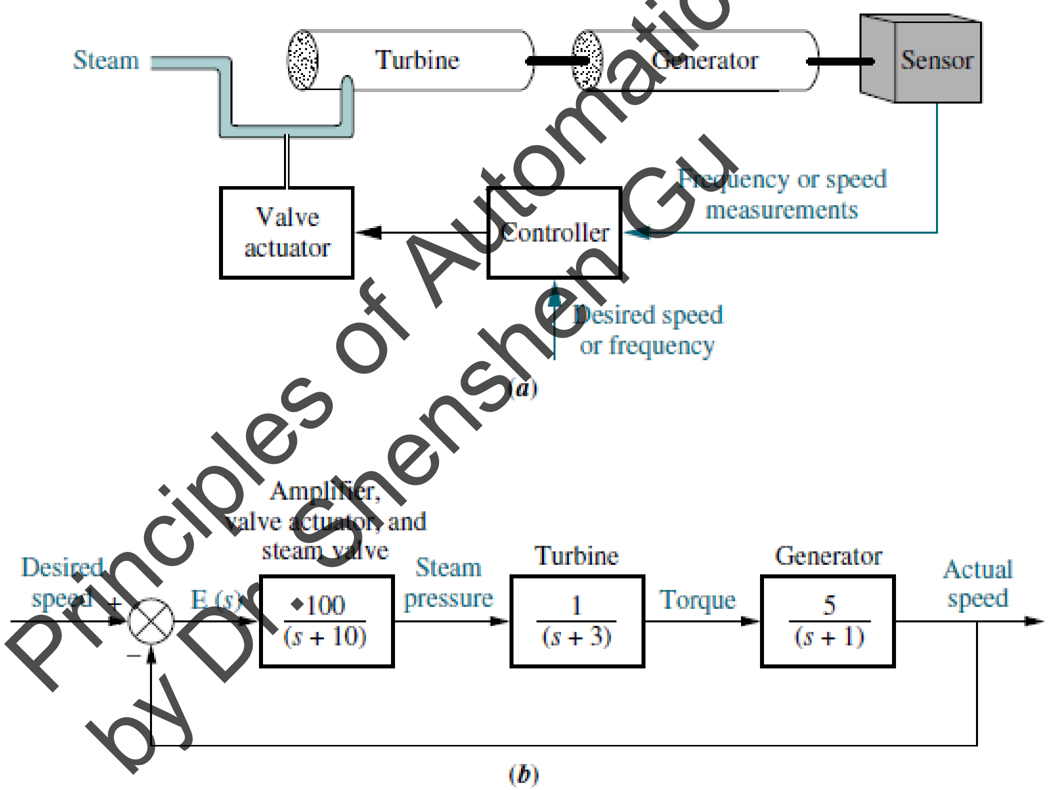
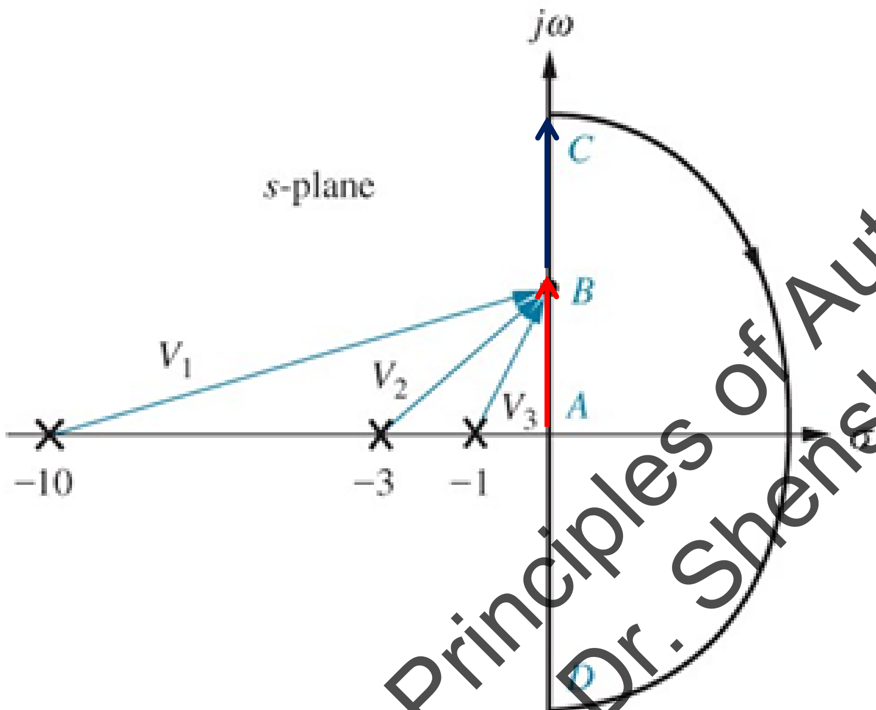
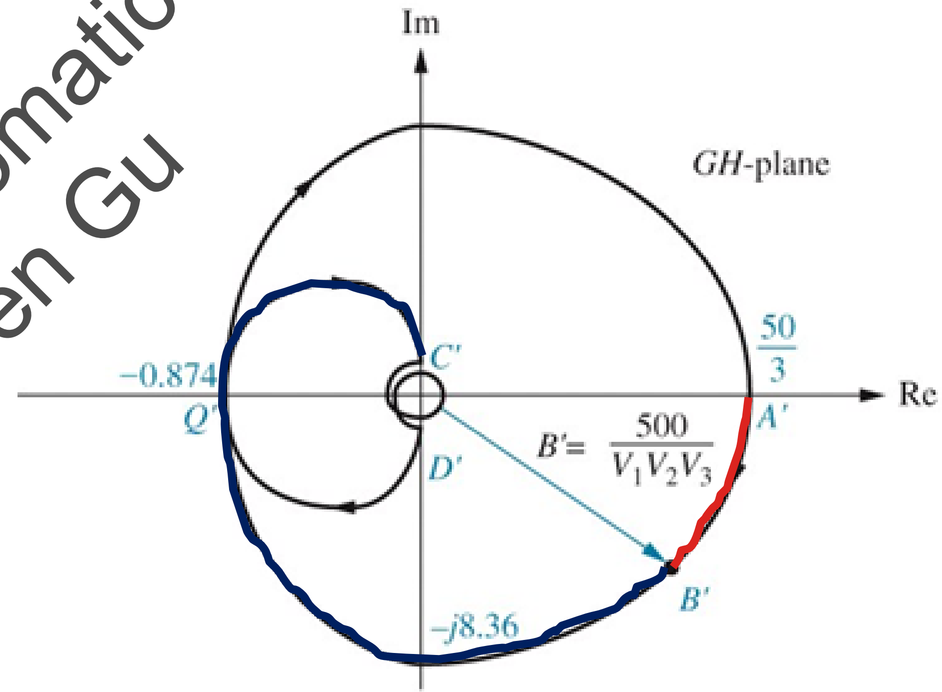
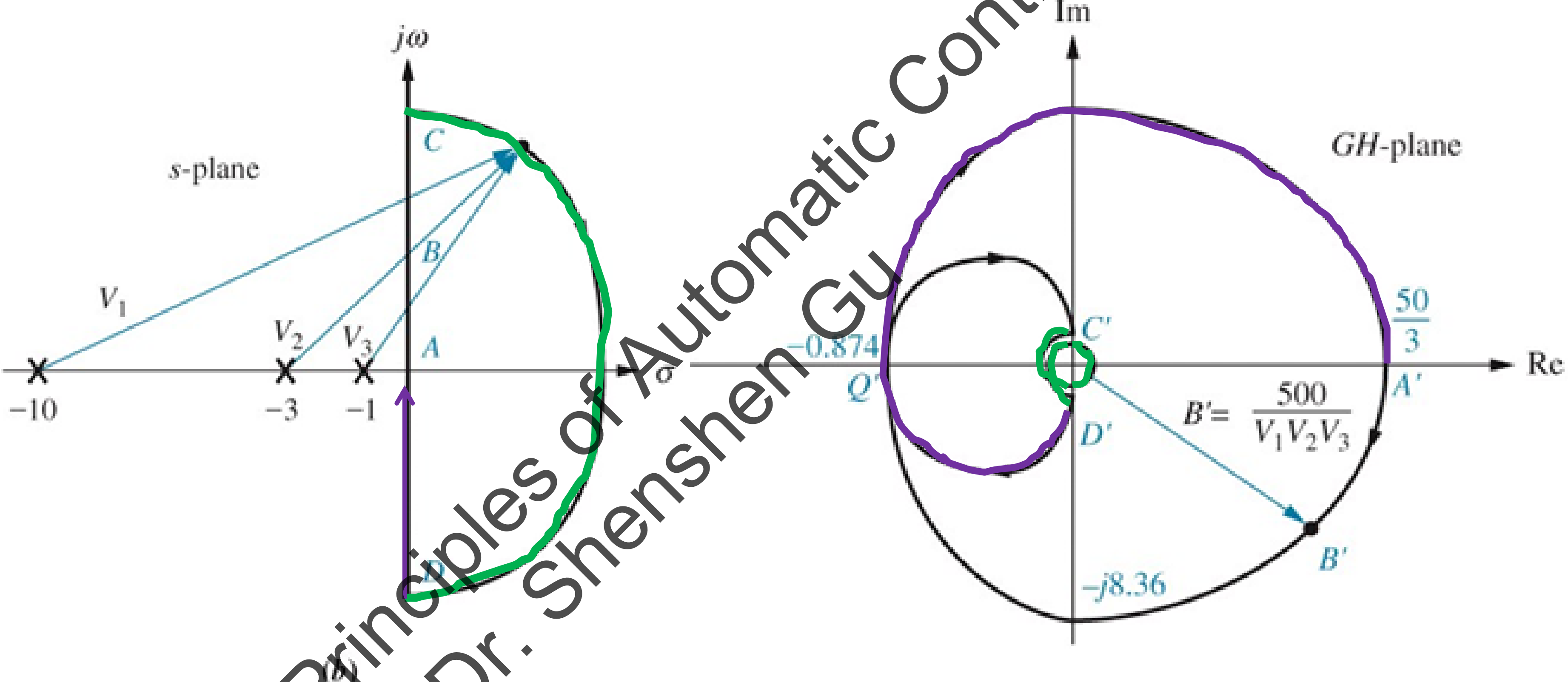


FIGURE 10.26
a. Turbine and generator;
b. block diagram of speed control system for Example 10.4

$$G(s)=\frac{500}{(s+1)(s+3)(s+10)}$$







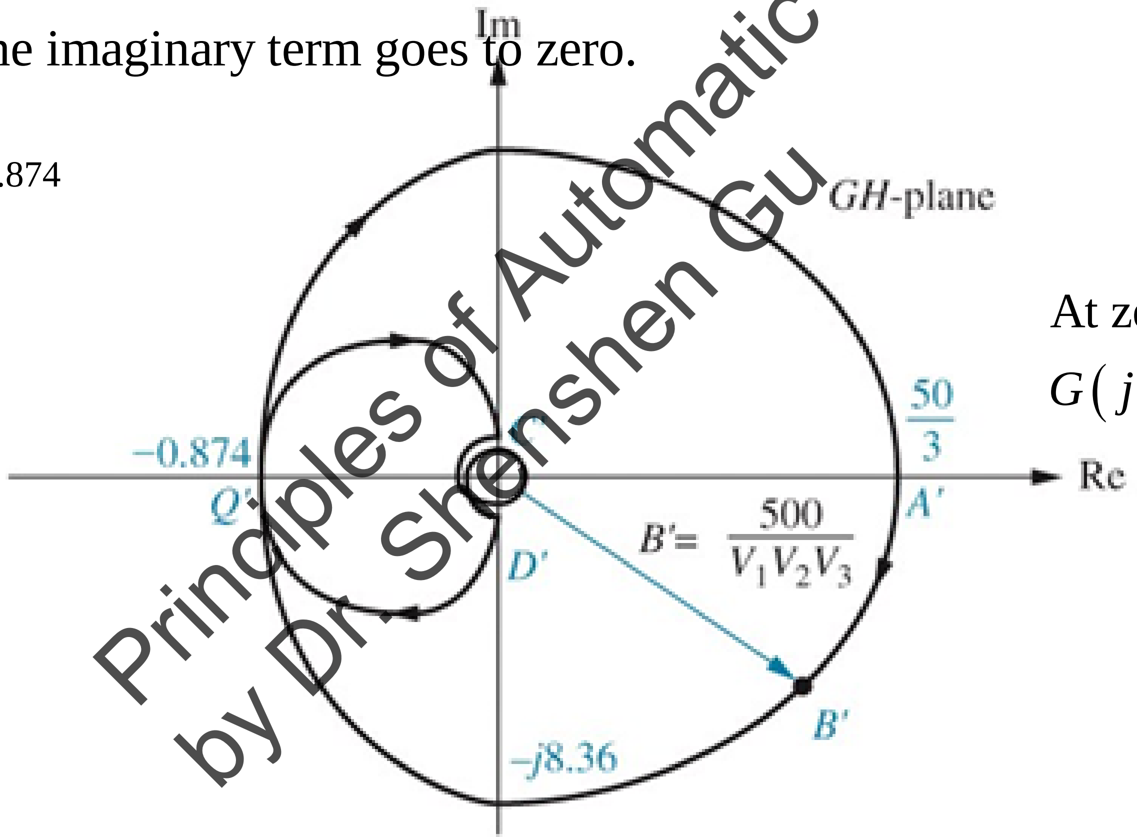
jω
s-plane
C
B
A
V_1
V_2
V_3
−10
−3
−1
σ
D
(b)
Im
GH-plane
C′
−0.874
Q′
D′
$\frac{50}{3}$
A′
Re
$B' = \frac{500}{V_1V_2V_3}$
B′
−j8.36

$$G(j\omega)=\frac{500}{(s+1)(s+3)(s+10)}\bigg|_{s=j\omega}=\frac{500}{(-14\omega^2+30)+j(43\omega-\omega^3)}$$

$$G(j\omega)=500\frac{(-14\omega^2+30)-j(43\omega-\omega^3)}{(-14\omega^2+30)^2+(43\omega-\omega^3)^2}$$

At $\omega=\sqrt{43}$, the Nyquist diagramcrosses the negative real axis since the imaginary term goes to zero.

$$G(j\omega)\big|_{\omega=\sqrt{43}}=-0.874$$



At zero frequency,

$$G(j\omega)=500/30=50/3$$

Skill-Assessment Exercise 10.3

PROBLEM: Sketch the Nyquist diagram for the system shown in Figure 10.10 where

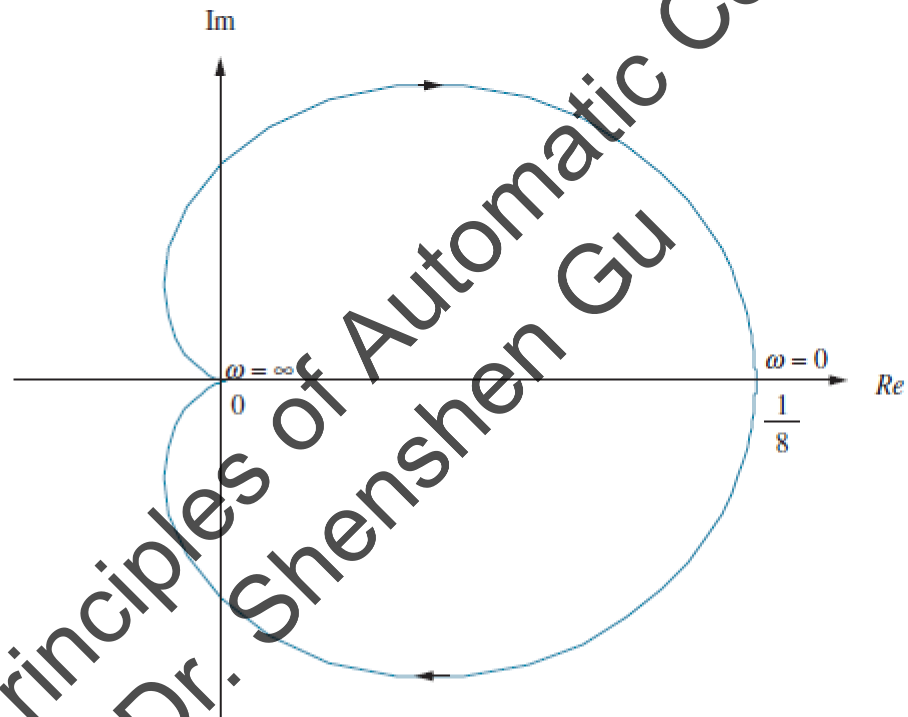
$$G(s) = \frac{1}{(s+2)(s+4)}$$

Compare your sketch with the polar plot in Skill-Assessment Exercise 10.1(c).

ANSWER: The complete solution is located at www.wiley.com/college/nise.

10.3

The frequency response is 1/8 at an angle of zero degrees at $\omega = 0$. Each pole rotates 90° in going from $\omega = 0$ to $\omega = \infty$. Thus, the resultant rotates -180° while its magnitude goes to zero. The result is shown below.



- In the previous example, there were no open-loop poles situated along the contour enclosing the right half-plane.
- If such poles exist, then a detour around the poles on the contour is required;
- otherwise, the mapping would go to infinity in an undetermined way, without angular information.

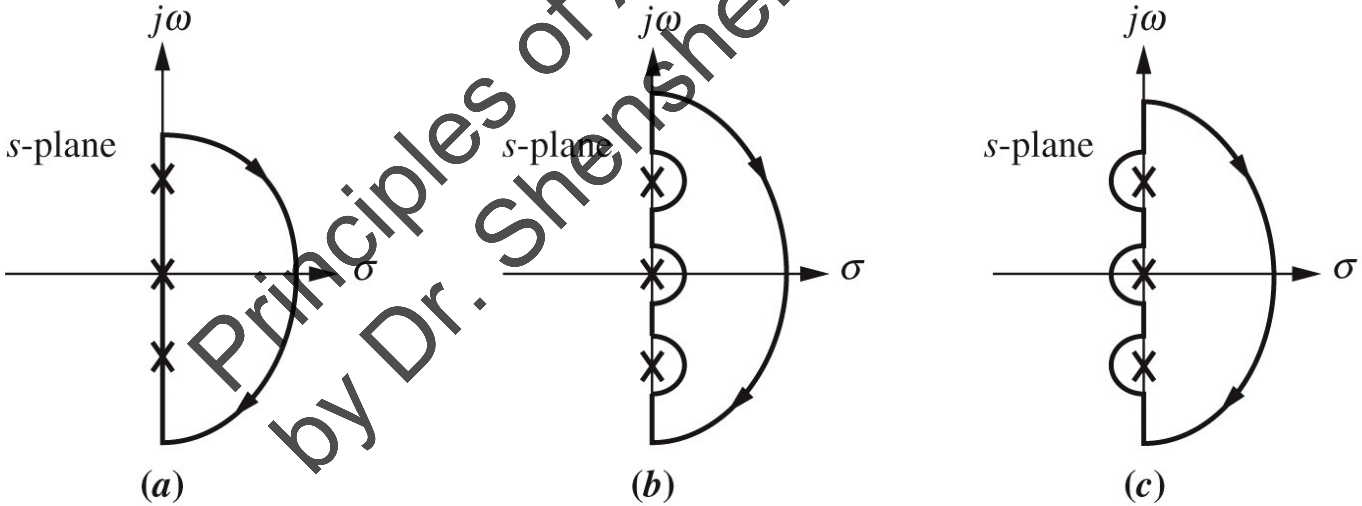


Figure 10.28

Example 10.5

Nyquist Diagram for Open-Loop Function with Poles on Contour

PROBLEM: Sketch the Nyquist diagram of the unity feedback system of Figure 10.10, where $G(s) = (s+2)/s^2$.

SOLUTION: The system's two poles at the origin are on the contour and must be bypassed, as shown in Figure 10.29(a). The mapping starts at point A and continues in a clockwise direction. Points A, B, C, D, E, and F of Figure 10.29(a) map respectively into points A', B', C', D', E', and F' of Figure 10.29(b).

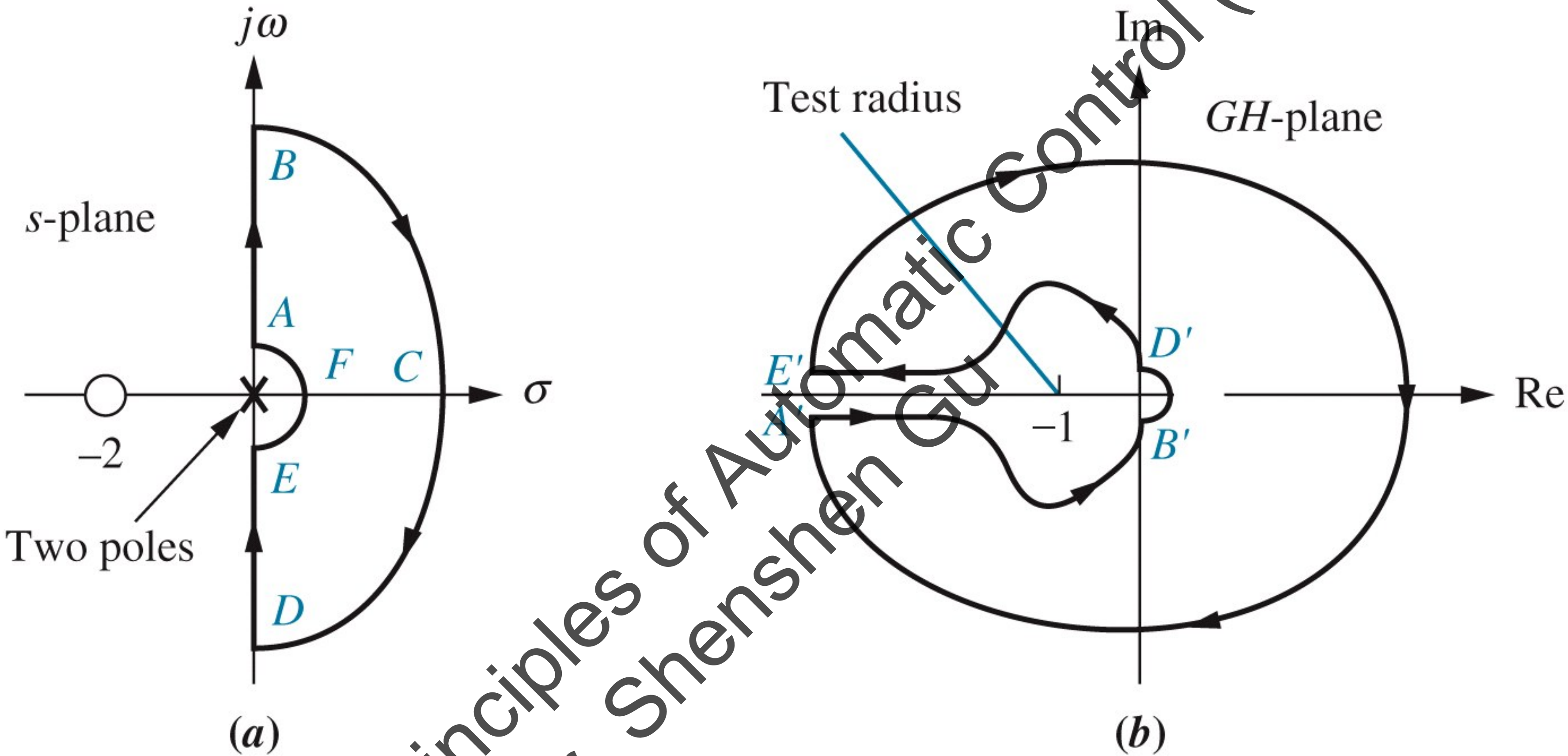

jω
s-plane
B
A
F
C
σ
–2
E
Two poles
D
(a)
Im
Test radius
GH-plane
D'
E'
A'
–1
B'
Re
(b)

Figure 10.29

Gain Margin and Phase Margin via the Nyquist Diagram

- Now that we know how to sketch and interpret a Nyquist diagram to determine a closed-loop system's stability, let us extend our discussion to concepts that will eventually lead us to the design of transient response characteristics via frequency response techniques.
- Using the Nyquist diagram, we define two quantitative measures of how stable a system is. These quantities are called gain margin and phase margin.
- Systems with greater gain and phase margins can withstand greater changes in system parameters before becoming unstable.

- Gain margin, (G_M) . The gain margin is the change in open-loop gain, expressed in decibels (dB), required at 180 of phase shift to make the closed-loop system unstable.
- Phase margin, (Φ_M). The phase margin is the change in open-loop phase shift required at unity gain to make the closed-loop system unstable.

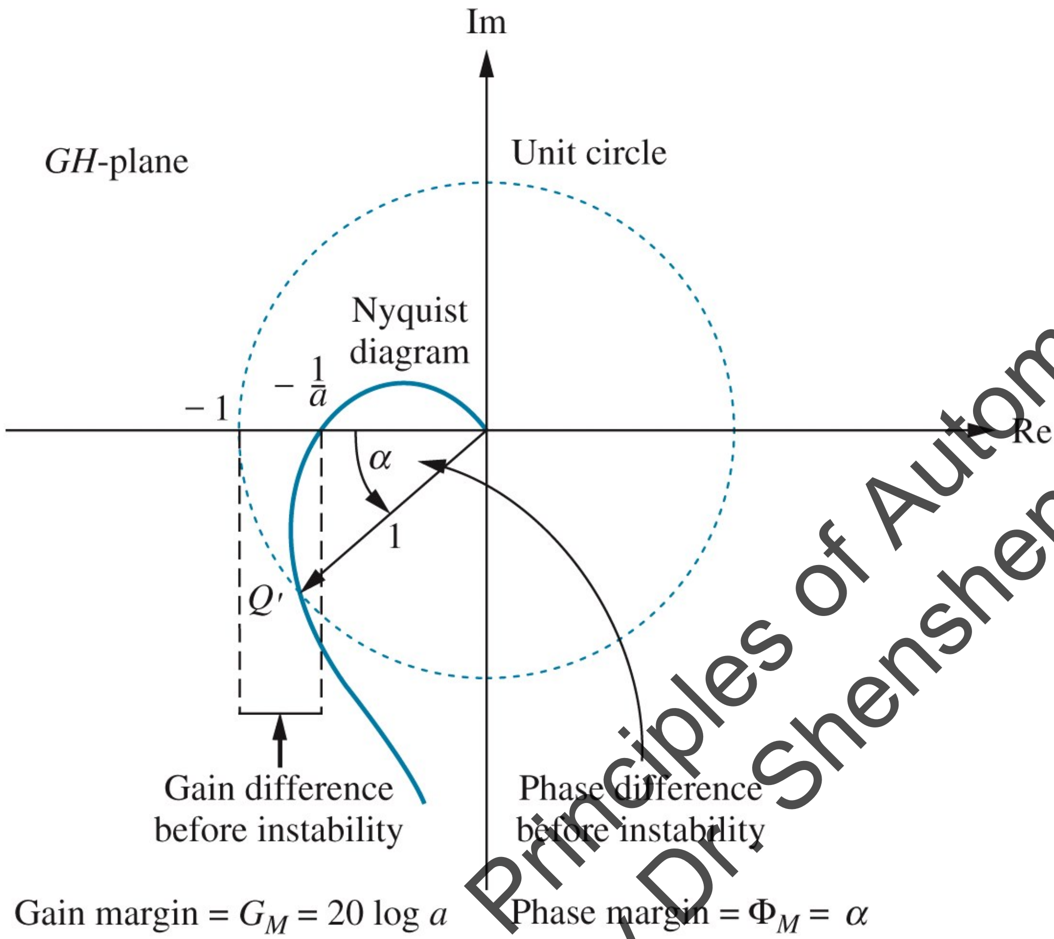


Figure 10.35

- Here a gain difference between the Nyquist diagram's crossing of the real axis at ***-1/a*** and the ***-1*** critical point determines the proximity of the system to instability. Thus, if the gain of the system were multiplied by a units, the Nyquist diagram would intersect the critical point. We then say that the ***gain margin*** is *a* units, or, expressed in dB, $G_M=20\log a$.
- At point Q', where the gain is unity, ***α*** represents the system's proximity to instability. That is, at unity gain, if a phase shift of ***α*** degrees occurs, the system becomes unstable. Hence, the amount of ***phase margin*** is ***α***.

- Later, we show that phase margin can be related to the damping ratio. Thus, we will be able to relate frequency response characteristics to transient response characteristics as well as stability. We will also show that the calculations of gain and phase margins are more convenient if Bode plots are used rather than a Nyquist diagram

Example 10.8

Finding Gain and Phase Margins

PROBLEM: Find the gain and phase margin for the system of Example 10.7 if $K = 6$.

SOLUTION: To find the gain margin, first find the frequency where the Nyquist diagram crosses the negative real axis. Finding $G(j\omega)H(j\omega)$, we have

$$G(j\omega)H(j\omega) = \left.\frac{6}{(s^2+2s+2)(s+2)}\right|_{s\to j\omega} = \frac{6[4(1-\omega^2) - j\omega(6-\omega^2)]}{16(1-\omega^2)^2 + \omega^2(6-\omega^2)^2} \tag{10.47}$$

The Nyquist diagram crosses the real axis at a frequency of $\sqrt{6}$ rad/s. The real part is calculated to be -0.3. Thus, the gain can be increased by $(1/0.3) = 3.33$ before the real part becomes -1. Hence, the gain margin is

$$G_M = 20 \log 3.33 = 10.45 \text{ dB} \tag{10.48}$$

To find the phase margin, find the frequency in Eq. (10.47) for which the magnitude is unity. As the problem stands, this calculation requires computational tools, such as a function solver or the program described in Appendix H.2. Later in the chapter we will simplify the process by using Bode plots. Eq. (10.47) has unity gain at a frequency of 1.253 rad/s. At this frequency, the phase angle is $-112.3°$. The difference between this angle and $-180°$ is $67.7°$, which is the phase margin.

Gain Margin, and Phase Margin via Bode Plots

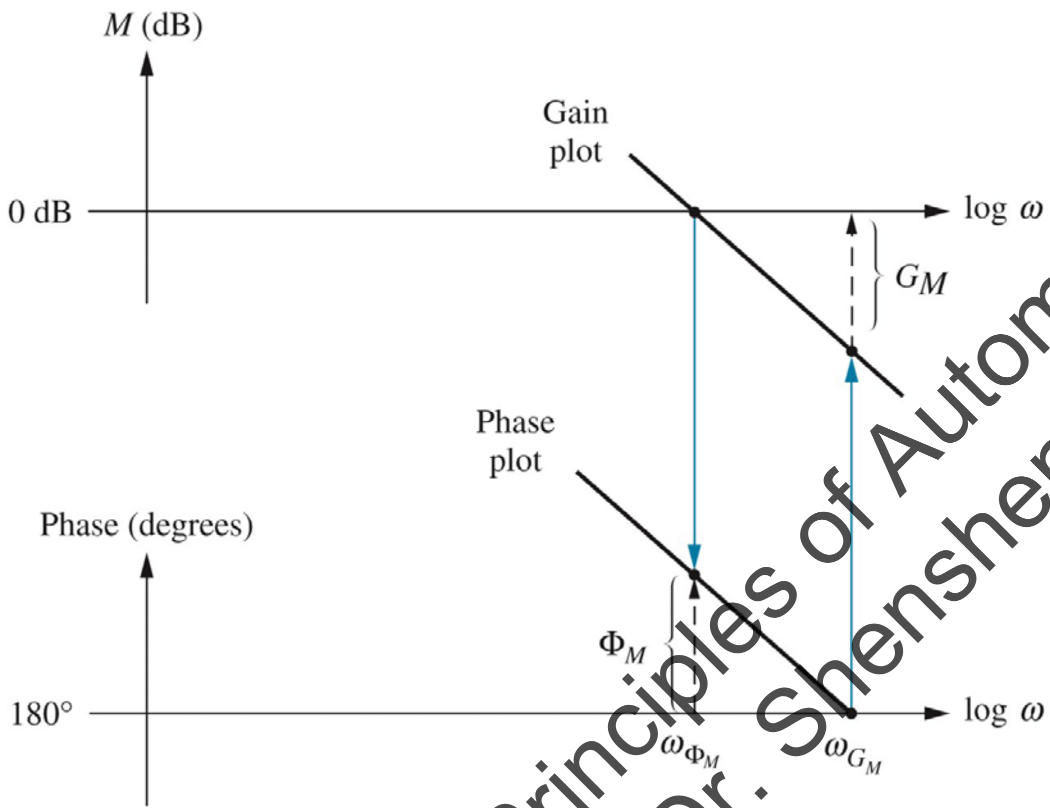


Figure 10.37

- The gain margin is found by using the phase plot to find the frequency, ω_{GM}, where the phase angle is 180. At this frequency, we look at the magnitude plot to determine the gain margin, G_M, which is the gain required to raise the magnitude curve to 0 dB.
- The phase margin is found by using the magnitude curve to find the frequency, $\omega_{\Phi M}$, where the gain is 0 dB. On the phase curve at that frequency, the phase margin, Φ_M, is the difference between the phase value and 180.

Relation Between Closed-Loop Transient and Open-Loop Frequency Responses

- Damping Ratio from Phase Margin
  - Let us now derive the relationship between the phase margin and the damping ratio. This relationship will enable us to evaluate the percent overshoot from the phase margin found from the open-loop frequency response.

Consider a unity feedback system whose open-loop function

$$\frac{\omega_n^2}{s\left(s+2\zeta\omega_n\right)} \xrightarrow{\text{Closed loop transfer function}} T(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

In order to evaluate the phase margin, we let

$$\left|G(j\omega)\right|=\frac{\omega_n^2}{\left|-\omega_n^2+j2\zeta\omega_n\omega\right|}=1 \quad\Rightarrow\quad \omega_1=\omega_n\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}$$

The phase angle at this frequency is

$$\angle G(j\omega)=-90-\tan^{-1}\frac{\omega_1}{2\zeta\omega_n}=-90-\tan^{-1}\frac{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}{2\zeta}$$

The difference between the angle and -180 is the phase margin

$$\Phi_M=90-\tan^{-1}\frac{\omega_1}{2\zeta\omega_n}=\tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}$$

This figure shows the relationship between phase margin and damping ratio

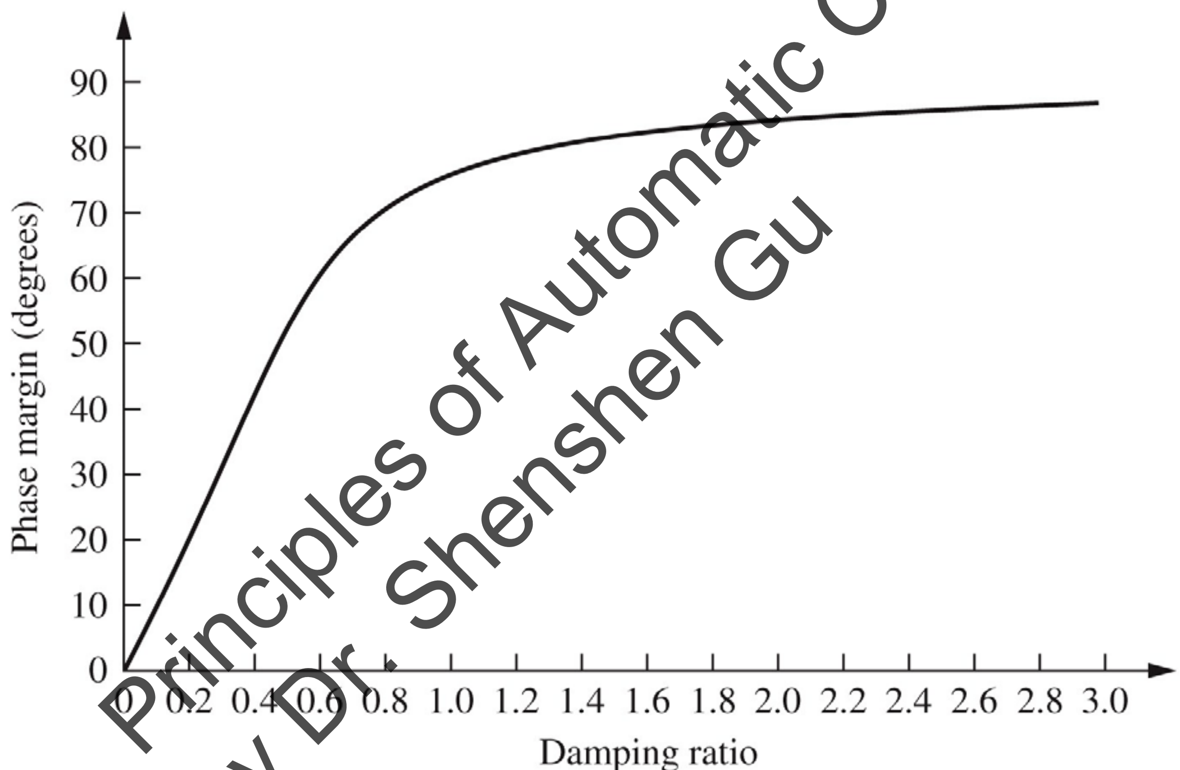
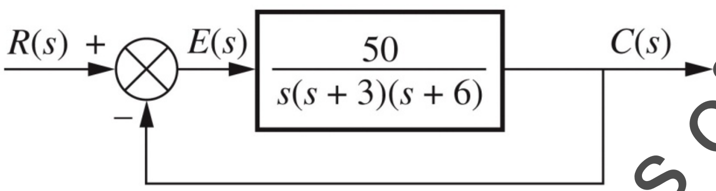


Figure 10.48

Example 10.13

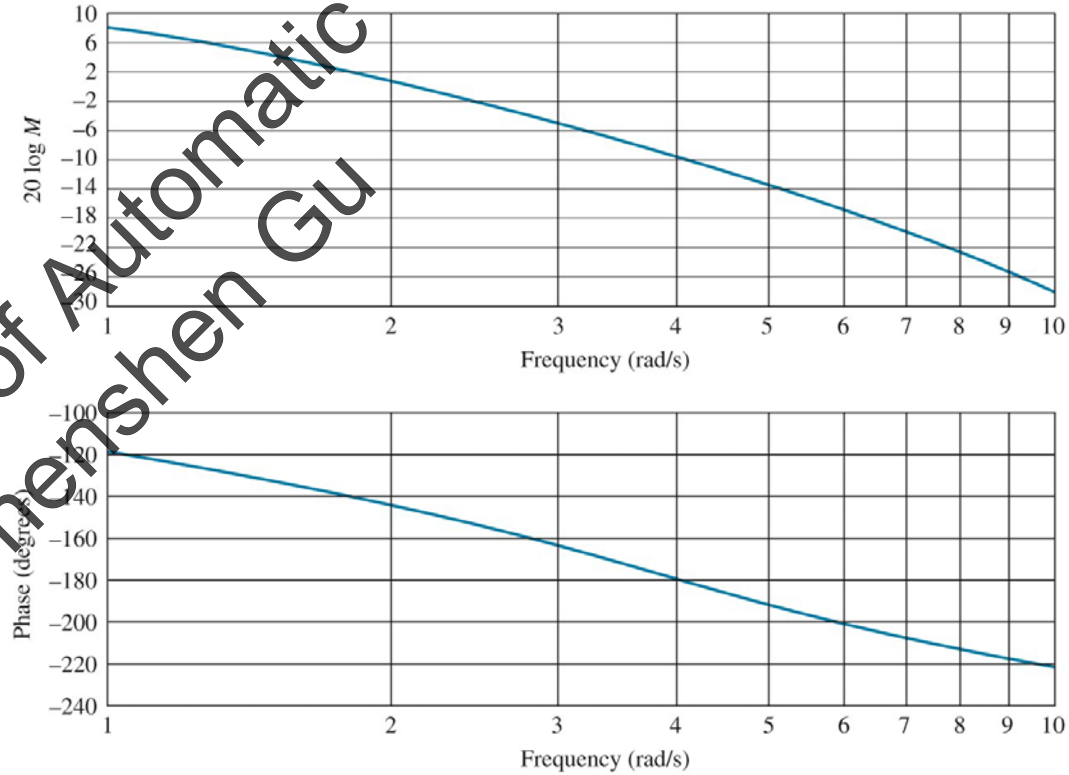
Settling and Peak Times from Open-Loop Frequency Response

PROBLEM: Given the system of Figure 10.50(a) and the Bode diagrams of Figure 10.50(b), estimate the settling time and peak time.



(*a*)

Figure 10.50a



(*b*)

Figure 10.50b

SOLUTION: Using Figure 10.50(b), we estimate the closed-loop bandwidth by finding the frequency where the open-loop magnitude response is in the range of -6 to -7.5 dB if the phase response is in the range of $-135°$ to $-225°$. Since Figure 10.50(b) shows -6 to -7.5 dB at approximately 3.7 rad/s with a phase response in the stated region, $\omega_{BW} \cong 3.7$ rad/s.

Next find ζ via the phase margin. From Figure 10.50(b), the phase margin is found by first finding the frequency at which the magnitude plot is 0 dB. At this frequency, 2.2 rad/s, the phase is about $-145°$. Hence, the phase margin is approximately $(-145° - (-180°)) = 35°$. Using Figure 10.48, $\zeta = 0.32$. Finally, using Eqs. (10.55) and (10.56), with the values of ω_{BW} and ζ just found, $T_s = 4.86$ seconds and $T_p = 1.29$ seconds. Checking the analysis with a computer simulation shows $T_s = 5.5$ seconds, and $T_p = 1.43$ seconds.

Summary

- Frequency response methods are an alternative to the root locus for analyzing and designing feedback control systems.
- Frequency response techniques can be used more effectively than transient response to model physical systems in the laboratory.
- From this data the magnitude frequency response of the system, which is the ratio of the output amplitude to the input amplitude, can be plotted and used in place of an analytically obtained magnitude frequency response. Similarly, we can obtain the phase response by finding the difference between the output phase angle and the input phase angle at different frequencies.

Summary (Cont.)

- The polar plot of G(s)H(s) is known as a Nyquist diagram. Separate magnitude and phase diagrams, sometimes referred to as Bode plots. An advantage of Bode plots over the Nyquist diagram is that they can easily be drawn using asymptotic approximations to the actual curve.
- The Nyquist criterion sets forth the theoretical foundation from which the frequency response can be used to determine a system's stability. Using the Nyquist criterion and Nyquist diagram, or the Nyquist criterion and Bode plots, we can determine a system's stability.