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Principles of Automatic Control (1)

自动控制原理1

Topic 5
Stability

(Chapter 6 in text book)

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Review for the previous topics

- Three objectives in designing a control system (Topic 1):
  - Transient response (Topic 3);
  - Stability (This topic);
  - Steady state error (Next topic).
- Stability is the most important system specification. If a system is unstable, transient response and steady-state errors are moot points.
- In this topic, we will study how to determine whether a system is stable or not.

New terminologies in this topic

- Stability 稳定性
- Stable 稳定的
- Unstable 不稳定的
- Bounded 有界的
- Left half-plane (lhp) 左半平面
- Right half-plane (rhp) 右半平面
- Routh-Hurwitz criterion 劳斯判据
- Routh table 劳斯表
- Row 行
- Column 列
- Determinant 行列式
- Auxiliary polynomial 辅助多项式
- Symmetrical 对称的
- Quadrantal symmetrical 象限对称的

Learning Outcomes for Topic 5

After completing this topic, you will be able to:

- Make and interpret a basic Routh table to determine the stability of a system;
- Make and interpret a Routh table where either the first element of a row is zero or an entire row is zero.

Outline

- Brief Introduction
- Routh-Hurwitz Criterion
- Routh-Hurwitz Criterion: Special Cases
  - Zero Only in the First Column
  - Entire Row is Zero
- Routh-Hurwitz Criterion: Additional Examples
- Summary

Brief Introduction

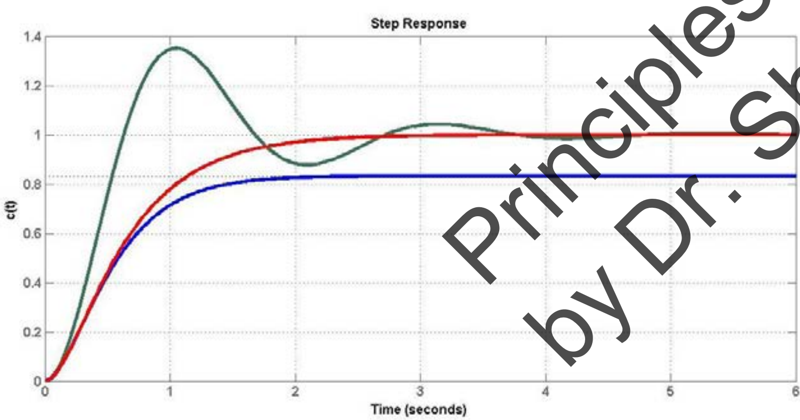
- Three objectives in designing a control system (Topic 1):
  - Transient response (Topic 3);
  - Stability (This topic);
  - Steady state error (Next topic).
- Stability is the most important system specification. If a system is unstable, transient response and steady-state errors are moot points.
- If an engineer makes a mistake in his stability analysis, and what he think is a stable system is actually unstable:
  - Unexpected unbounded system response;
  - Damage to property;
  - Injury or death to people in the vicinity ;

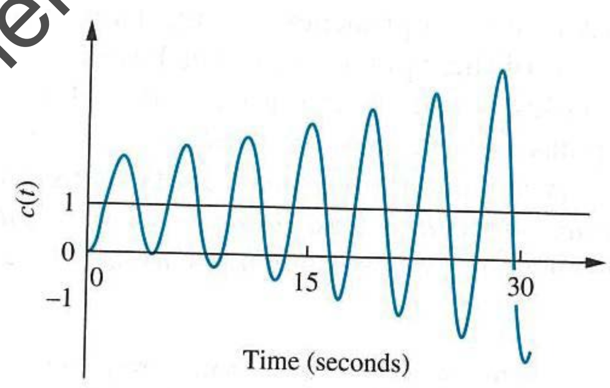
What is stability?

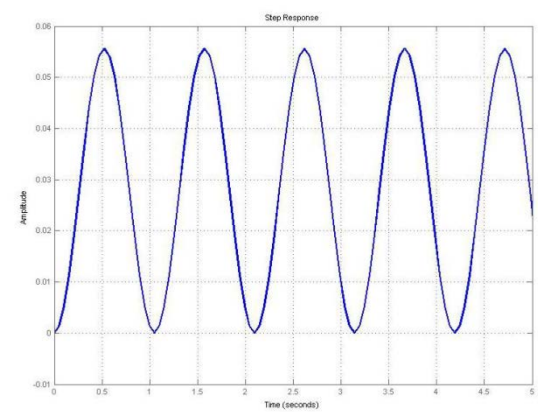
- There are many definitions for stability, depending upon the kind of system or the point of view.
  - Stability definition for linear systems from the viewpoint of natural response;
  - Stability definition for linear systems from the viewpoint of total response.

Stability definition for linear systems from the viewpoint of natural response

- Total response = Natural response + Forced response
  - Stable: Natural response decays to zero as time approaches infinity;
  - Unstable: Natural response increases without bound;
  - Marginally stable: Natural response neither decay nor grow without bound but oscillate.
- These definitions rely on a description of the natural response.
- It may be difficult to separate the natural response from the forced response.





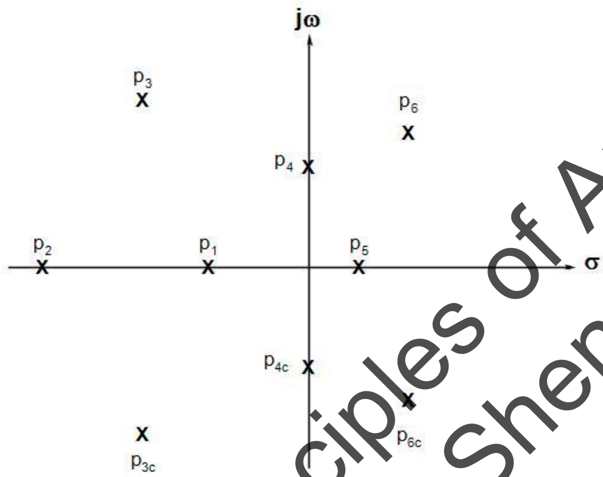


Stability definition for linear systems from the viewpoint of total response

- If the input is bounded and the total response is not approaching infinity as time approaches infinity, then the natural response is obviously not approaching infinity.
  - Stable: If every bounded input yields a bounded output (BIBO);
  - Unstable: If any bounded input yields an unbounded output.

How to determine whether a system is stable or not?

- Focus on the natural response definitions of stability.
- ***The poles of the transfer function*** generate the form of the ***natural response***. (Topic 3)



p_1 & p_2 = Exponential decay
p_3 & p_{3c} = Decaying oscillation
p_4 & p_{4c} = Pure oscillation (marginal)

p_5 = Exponential increasing
p_6 & p_{6c} = Increasing oscillation

- Stable systems have closed-loop transfer functions with poles ONLY in the left half-plane;
- Unstable systems have closed-loop transfer functions with at least one pole in the right half-plane and/or poles of multiplicity greater than 1 on the imaginary axis.
- Marginally stable systems have closed-loop transfer functions with only imaginary axis poles of multiplicity 1 and poles in the left half-plane.

Example of Stable System

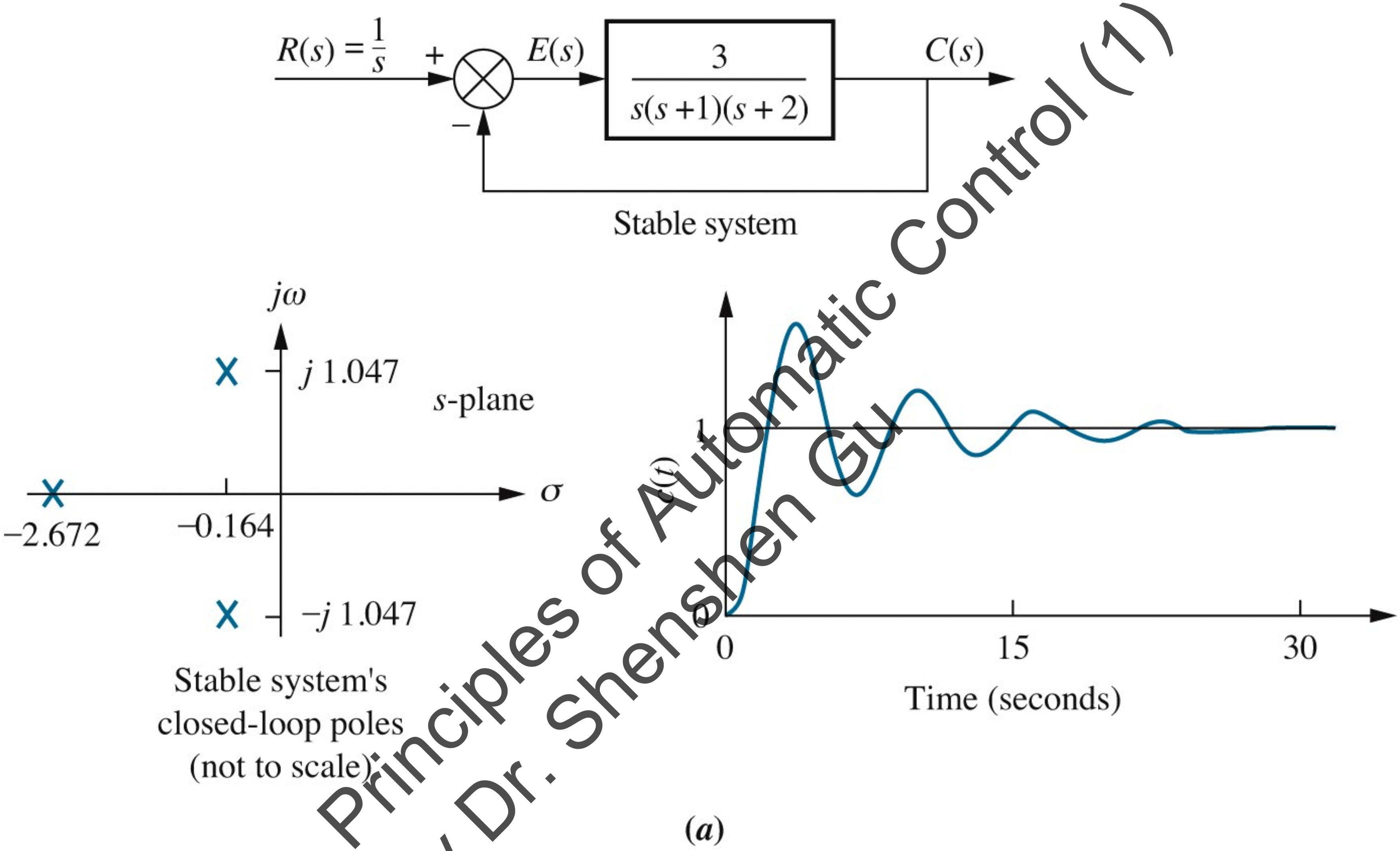


Figure 6.1a

Example of Unstable System

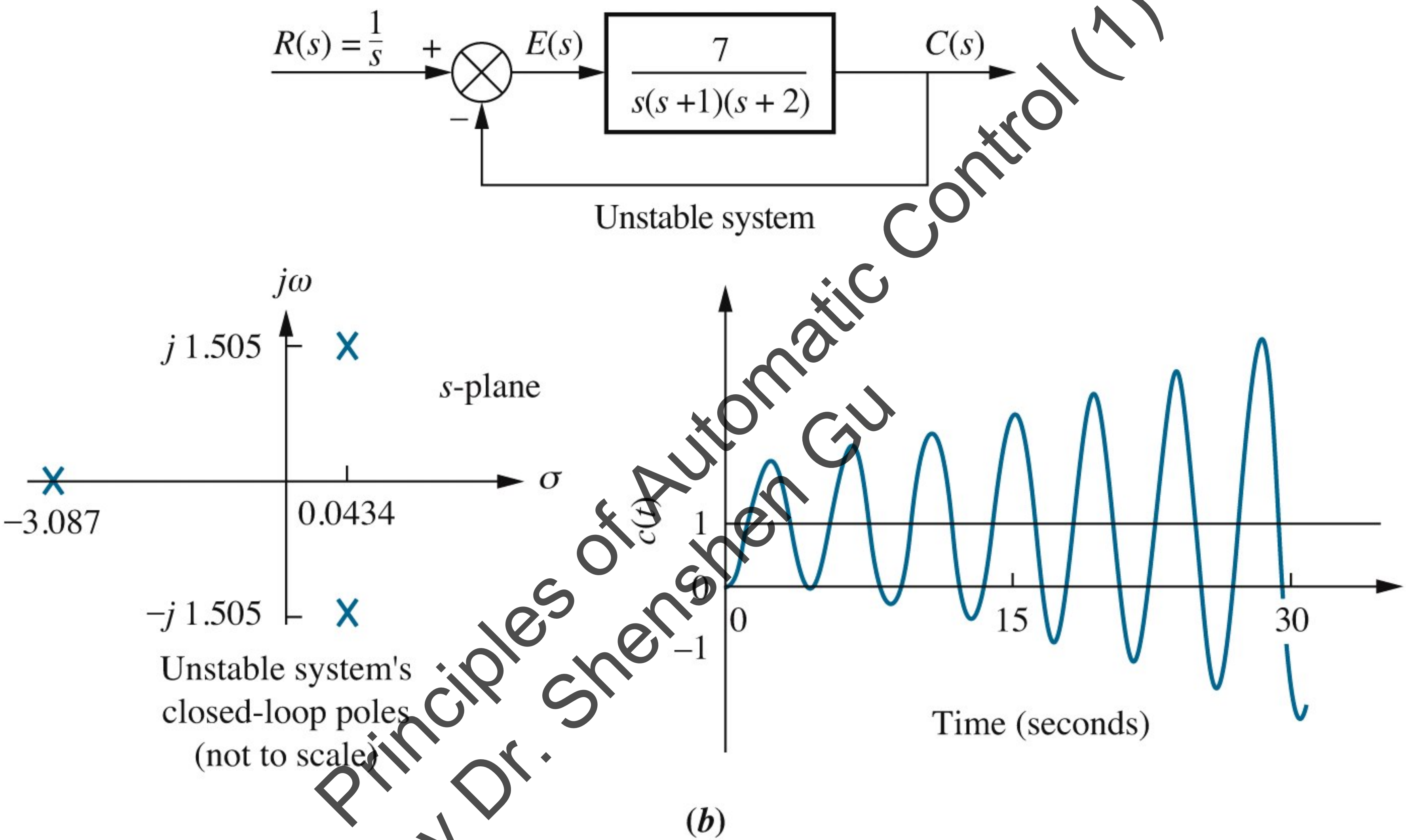


Figure 6.1b

- It is not always a simple matter to determine if a feedback control system is stable.
- We know the poles of the forward transfer function in the following system, but we do not know the location of the poles of the equivalent closed-loop system.

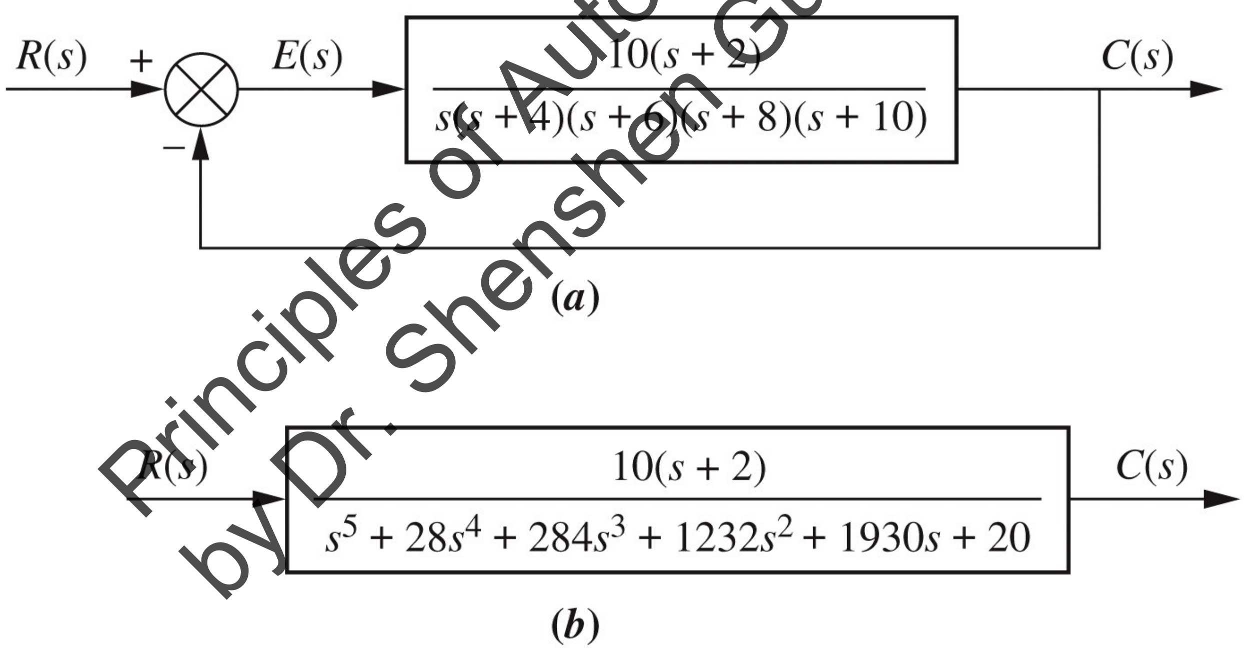


Figure 6.2

- What do you do when the closed loop transfer function polynomial is horrendous?

$$s^{234} + 54.32s^{233} + \cdots + 0.0032s + 4$$

- One option – plug into a calculator or Matlab to solve for poles.
- Good for checking a **specific** system configuration, but won't give you a **range** of allowable system parameters.

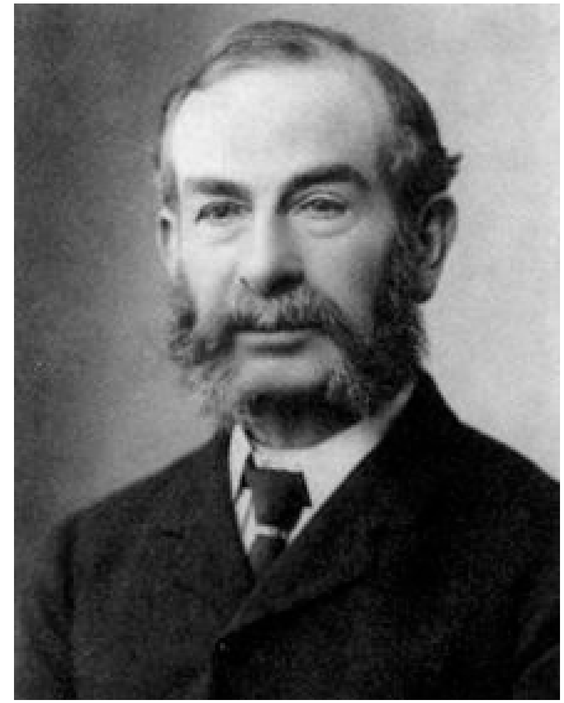
- Example where you have an unknown Spring constant K in the denominator polynomial:

$$s^{234} + Ks^{233} + \cdots + 0.0032s + 4$$

- Use trial and error values of *K* to find stable system configurations, but could be painful!
- Solution – the Routh-Hurwitz criterion for stability.
- This is a method for finding out ***how many*** closed-loop system poles are in the left half plane, right half plane, and on the imaginary axis.
- Doesn't tell us ***where*** the poles are located, but this doesn't matter for simply working out whether a system is ***stable or unstable***.

Routh-Hurwitz Criterion

- Routh, E.J. *Dynamics of a System of Rigid Bodies*, 6th ed. Macmillam, London, 1905
- This method requires two steps:
- Generate a data table called a ***Routh table***;
- Interpret the Routh table to tell how many closed-loop system poles are in the left half-plane, the right half-plane, and on the *jω*-axis.
- **The power of the method lies in design rather than analysis.**



Edward John Routh (1831-1907)

Generating a Basic Routh Table

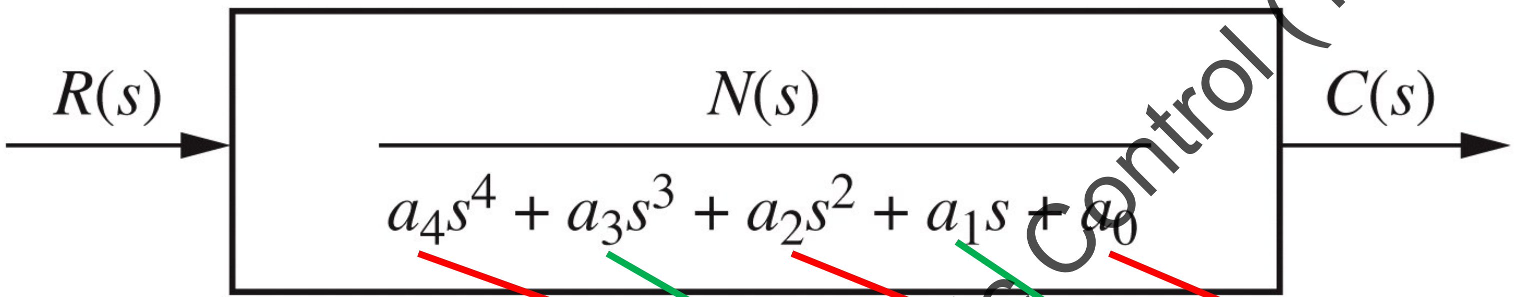


Figure 6.3

- Begin by labeling the rows with powers of s from the highest power of the demoninator of the closed-loop transfer function to s^0.
- Next start with the coefficient of the highest power of s in the denominator and list, horizontally in the first row, every other coefficient.
- In the second row, list horizontally, starting with the next highest power of s, every coefficient that was skipped in the first row.

TABLE 6.1 Initial layout for Routh table

Table 6.1

TABLE 6.2 Completed Routh table

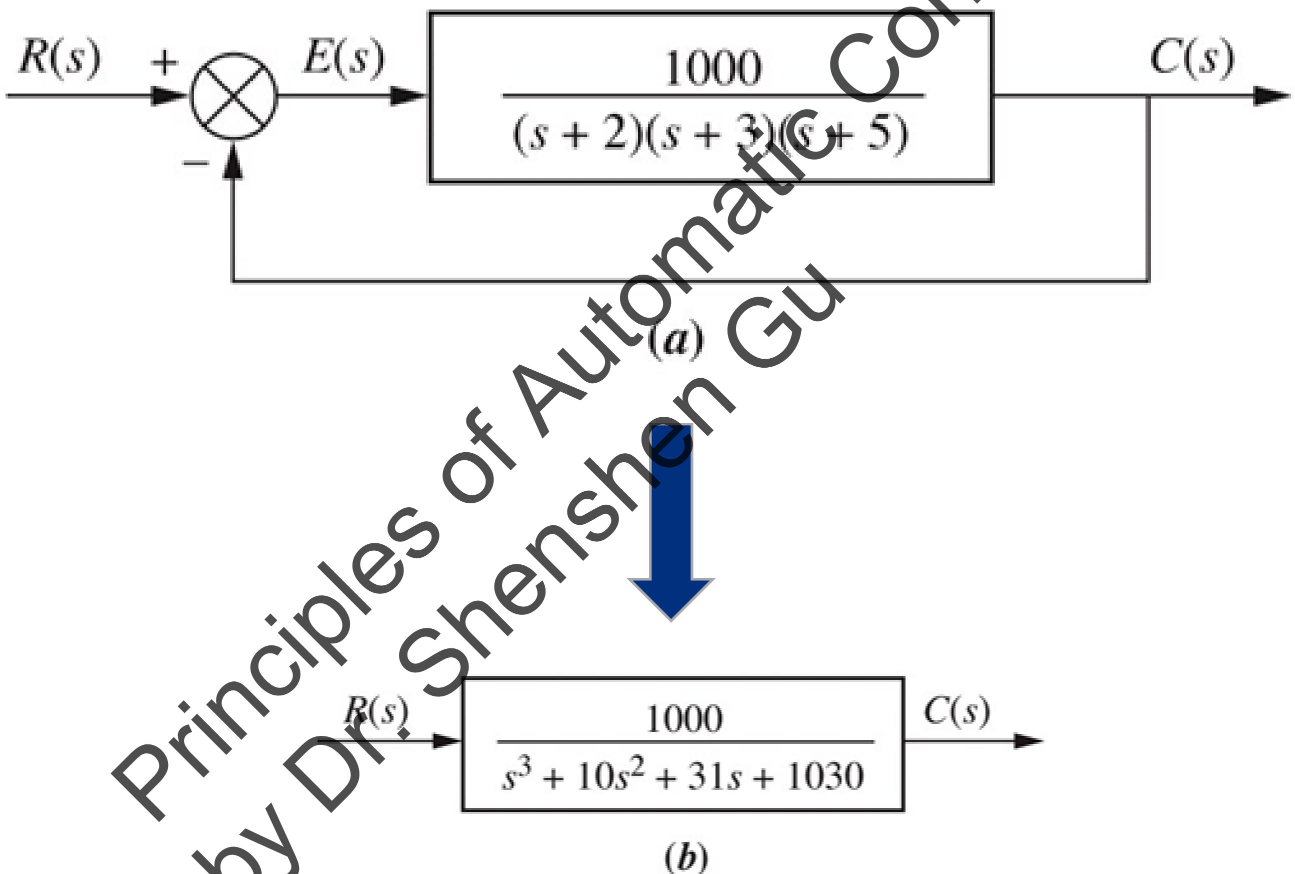
s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

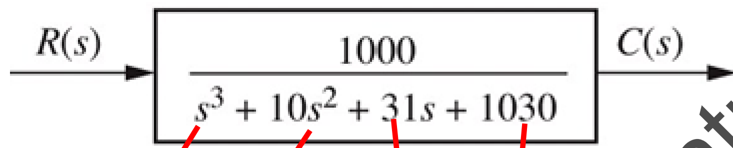
Table 6.2

- The remaining entries are filled in as follows.
  - Each entry is a negative determinant of entries in the previous two rows divided by the entry in the first column directly above the calculated row.
  - The left-hand column of the determinant is always the first column of the previous two rows, and the right-hand column is the elements of the column above and to the right.
  - The table is complete when all of the rows are completed down to s^0.

Example

- Make the Routh table for the system shown in the following figure:





(b)

For convenience, any row of the Routh table can be multiplied by a positive constant without changing the values of the rows below.

s^3	1	31	0
s^2	10 x 0.1 = 1	1030 x 0.1 = 103	0
s^1	$\dfrac{-\begin{vmatrix} 1 & 31 \\ 1 & 103 \end{vmatrix}}{1} = -72$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0$	
s^0	$\dfrac{-\begin{vmatrix} 1 & 103 \\ -72 & 0 \end{vmatrix}}{-72} = 103$		

$$G(S)=\frac{1000}{S^3+10S^2+31S+1030}$$

s^3	1	31	0
s^2	1	103	0
s^1	-72	0	0
s^0	103	0	0

Interpreting the Basic Routh Table

- **The Routh-Hurwitz criterion declares that the number of roots of the polynomial that are in the right half-plane is equal to the number of sign changes in the first column.**

$$G(S) = \frac{1000}{S^3 + 10S^2 + 31S + 1030}$$

s^3	1	31	0
s^2	1	103	0
s^1	-72	0	0
s^0	103	0	0

Thus, the system is unstable since two poles exist in the right half-plane.

```
roots([1,10,31,1030])
    -13.4136
     1.7068 + 8.5950i
     1.7068 - 8.5950i
```

Skill-Assessment Exercise 6.1

WileyPLUS
WPCS
Control Solutions

PROBLEM: Make a Routh table and tell how many roots of the following polynomial are in the right half-plane and in the left half-plane.

$$P(s) = 3s^7 + 9s^6 + 6s^5 + 4s^4 + 7s^3 + 8s^2 + 2s + 6$$

ANSWER: Four in the right half-plane (rhp), three in the left half-plane (lhp).

The complete solution is at www.wiley.com/college/nise.

6.1

Make a Routh table.

s^7	3	6	7	2
s^6	9	4	8	6
s^5	4.666666667	4.333333333	0	0
s^4	−4.35714286	8	6	0
s^3	12.90163934	6.426229508	0	0
s^2	10.17026684	6	0	0
s^1	−1.18515742	0	0	0
s^0	6	0	0	0

1 2 3 4

One minute Quiz:

(1) Is this system stable or unstable?

(A) Stable (B) Unstable √

(2) How many roots located in the right half plane?

(A) One (B) Two (C) Three (D) Four √

Special Cases 1: Zero Only in the First Column

- If the first element of a row is zero, division by zero would be required to form the next row.
- To avoid this phenomenon, an extremely small positive number, ε, is assigned to replace the zero in the first column.

Example: Determine the stability of the closed-loop transfer function $T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$

s^5	1	3	5
s^4	2	6	3
s^3	$\not{0}$ ϵ	$\frac{7}{2}$	0
s^2	$\frac{6\epsilon - 7}{\epsilon}$	3	0
s^1	$\frac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14}$	0	0
s^0	3	0	0

Label	First column	$\epsilon = +$
s^5	1	$+$
s^4	2	$+$
s^3	~~0~~ ϵ	$+$
s^2	$\frac{6\epsilon - 7}{\epsilon}$	$-$
s^1	$\frac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14}$	$+$
s^0	3	$+$

The table shows a sign change from the s^3 row to the s^2 row, and there will be another sign change from the s^2 row to the s^1 row. Hence, the system is unstable and has two poles in the right half-plane.

Special Cases 2: Entire Row is Zero

- Sometimes while making a Routh table, we find that an entire row consists of zeros because there is an even polynomial that is a factor of the original polynomial.
- This case must be handled differently from the case of a zero in only the first column of a row.

Example 6.4

Stability via Routh Table with Row of Zeros

PROBLEM: Determine the number of right-half-plane poles in the closed-loop transfer function

$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56} \tag{6.8}$$

SOLUTION: Start by forming the Routh table for the denominator of Eq. (6.8) (see Table 6.7). At the second row we multiply through by 1/7 for convenience. We stop at the third row, since the entire row consists of zeros, and use the following

TABLE 6.7 Routh table for Example 6.4

s^5			1			6			8
s^4		~~7~~	1		~~42~~	6		~~56~~	8
s^3	~~0~~	~~4~~	1	~~0~~	~~12~~	3	~~0~~	~~0~~	0
s^2			3			8			0
s^1			$\frac{1}{3}$			0			0
s^0			8			0			0

procedure. First we return to the row immediately above the row of zeros and form an auxiliary polynomial, using the entries in that row as coefficients. The polynomial will start with the power of s in the label column and continue by skipping every other power of s. Thus, the polynomial formed for this example is

$$P(s) = s^4 + 6s^2 + 8 \tag{6.9}$$

Next we differentiate the polynomial with respect to s and obtain

$$\frac{dP(s)}{ds} = 4s^3 + 12s + 0 \tag{6.10}$$

Finally, we use the coefficients of Eq. (6.10) to replace the row of zeros. Again, for convenience, the third row is multiplied by 1/4 after replacing the zeros.

The remainder of the table is formed in a straightforward manner by following the standard form shown in Table 6.2. Table 6.7 shows that all entries in the first column are positive. Hence, there are no right–half-plane poles.

- An entire row of zeros will appear in the Routh table when a purely even or purely odd polynomial is a factor of the original polynomial.

$$s^4 + 5s^2 + 7$$

- Even polynomials only have roots that are symmetrical about the origin. This symmetry can occur under three conditions of root position:
  - The roots are symmetrical and real;
  - The roots are symmetrical and imaginary;
  - The roots are quadrantal.

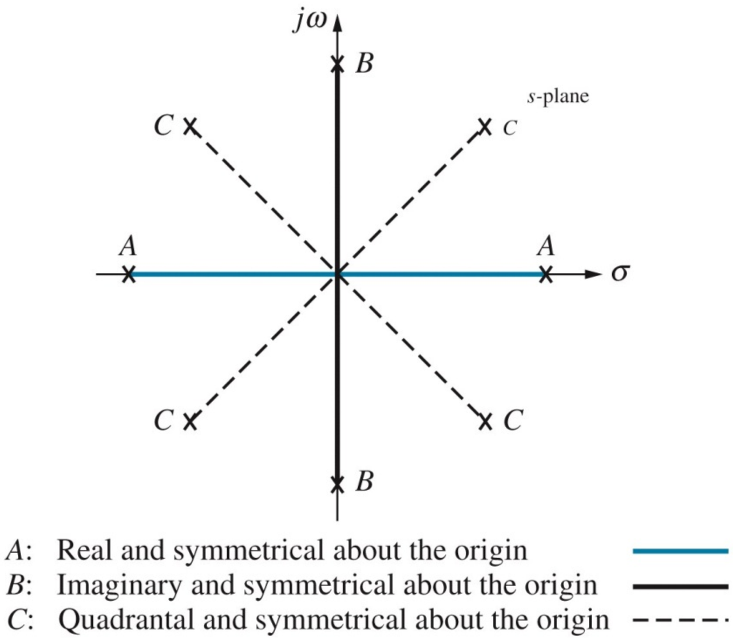


Figure 6.5

- It is this even polynomial that causes the row of zeros to appear.
- The row previous to the row of zeros contains the even polynomial that is a factor of the original polynomial. Everything from the row containing the even polynomial down to the end of the Routh table is a test of only the even polynomial.

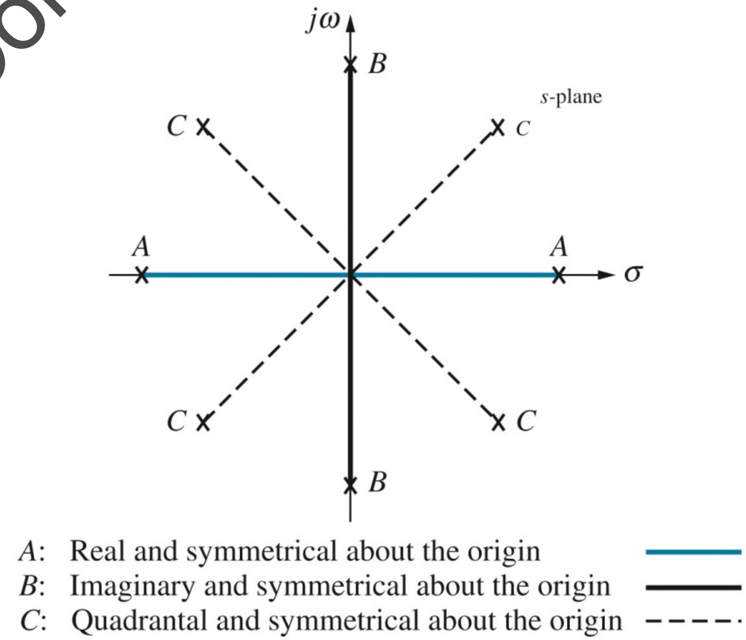


Figure 6.5

Example 6.5

Pole Distribution via Routh Table with Row of Zeros

PROBLEM: For the transfer function

$$T(s) = \frac{20}{s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20} \qquad (6.11)$$

tell how many poles are in the right half-plane, in the left half-plane, and on the $j\omega$-axis.

SOLUTION: Use the denominator of Eq. (6.11) and form the Routh table in Table 6.8. For convenience the s^6 row is multiplied by 1/10, and the s^5 row is multiplied by 1/20. At the s^3 row we obtain a row of zeros. Moving back one row to s^4, we extract the even polynomial, $P(s)$, as

$$P(s) = s^4 + 3s^2 + 2 \qquad (6.12)$$

TABLE 6.8 Routh table for Example 6.5

s^8	1	12	39	48	20
s^7	1	22	59	38	0
s^6	~~−10~~ −1	~~−20~~ −2	~~10~~ 1	~~20~~ 2	0
s^5	~~20~~ 1	~~60~~ 3	~~40~~ 2	0	0
s^4	1	3	2	0	0
s^3	~~0~~ ~~4~~ 2	~~0~~ ~~6~~ 3	~~0~~ ~~0~~ 0	0	0
s^2	~~$\frac{3}{2}$~~ 3	~~2~~ 4	0	0	0
s^1	$\frac{1}{3}$	0	0	0	0
s^0	4	0	0	0	0

This polynomial will divide evenly into the denominator of Eq. (6.11) and thus is a factor. Taking the derivative with respect to s to obtain the coefficients that replace the row of zeros in the s^3 row, we find

$$\frac{dP(s)}{ds} = 4s^3 + 6s + 0 \tag{6.13}$$

Replace the row of zeros with 4, 6, and 0 and multiply the row by 1/2 for convenience. Finally, continue the table to the s^0 row, using the standard procedure.

How do we now interpret this Routh table? Since all entries from the even polynomial at the s^4 row down to the s^0 row are a test of the even polynomial, we begin to draw some conclusions about the roots of the even polynomial. No sign changes exist from the s^4 row down to the s^0 row. Thus, the even polynomial does not have right–half-plane poles. Since there are no right–half-plane poles, no left–half-plane poles are present because of the requirement for symmetry. Hence, the even polynomial, Eq. (6.12), must have all four of its poles on the $j\omega$-axis.[4] These results are summarized in the first column of Table 6.9.

TABLE 6.9 Summary of pole locations for Example 6.5

	Polynomial		
Location	**Even (fourth-order)**	**Other (fourth-order)**	**Total (eighth-order)**
Right half-plane	0	2	2
Left half-plane	0	2	2
$j\omega$	4	0	4

The remaining roots of the total polynomial are evaluated from the s^8 row down to the s^4 row. We notice two sign changes: one from the s^7 row to the s^6 row and the other from the s^6 row to the s^5 row. Thus, the other polynomial must have two roots in the right half-plane. These results are included in Table 6.9 under "Other". The final tally is the sum of roots from each component, the even polynomial and the other polynomial, as shown under "Total" in Table 6.9. Thus, the system has two poles in the right half-plane, two poles in the left half-plane, and four poles on the $j\omega$-axis; it is unstable because of the right-half-plane poles.

Skill-Assessment Exercise 6.2

PROBLEM: Use the Routh-Hurwitz criterion to find how many poles of the following closed-loop system, $T(s)$, are in the rhp, in the lhp, and on the $j\omega$-axis:

$$T(s) = \frac{s^3 + 7s^2 - 21s + 10}{s^6 + s^5 - 6s^4 + 0s^3 - s^2 - s + 6}$$

ANSWER: Two rhp, two lhp, and two $j\omega$

The complete solution is at www.wiley.com/college/nise.

6.2

Make a Routh table. We encounter a row of zeros on the s^3 row. The even polynomial is contained in the previous row as $-6s^4 + 0s^2 + 6$. Taking the derivative yields $-24s^3 + 0s$. Replacing the row of zeros with the coefficients of the derivative yields the s^3 row. We also encounter a zero in the first column at the s^2 row. We replace the zero with ε and continue the table. The final result is shown now as

s^6	1	−6	−1	6	
s^5	1	0	−1	0	
s^4	−6	0	6	0	
s^3	−24	0	0	0	ROZ
s^2	ε	6	0	0	
s^1	$144/\varepsilon$	0	0	0	
s^0	6	0	0	0	

roots([1,1,-6,0,-1,-1,6])
-3.0000
2.0000
-1.0000
-0.0000 + 1.0000i
-0.0000 - 1.0000i
1.0000

There is one sign change below the even polynomial. Thus the even polynomial (4th order) has one right half-plane pole, one left half-plane pole, and 2 imaginary axis poles. From the top of the table down to the even polynomial yields one sign change. Thus, the rest of the polynomial has one right half-plane root, and one left half-plane root. The total for the system is two right half-plane poles, two left half-plane poles, and 2 imaginary poles.

Routh-Hurwitz Criterion: Additional Examples

Example 6.6

Standard Routh-Hurwitz

PROBLEM: Find the number of poles in the left half-plane, the right half-plane, and on the $j\omega$-axis for the system of Figure 6.6.

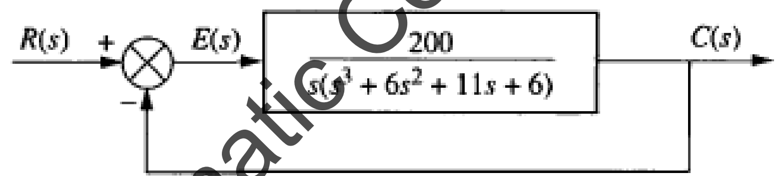


FIGURE 6.6 Feedback control system for Example 6.6

SOLUTION: First, find the closed-loop transfer function as

$$T(s) = \frac{200}{s^4 + 6s^3 + 11s^2 + 6s + 200} \tag{6.14}$$

The Routh table for the denominator of Eq. (6.14) is shown as Table 6.10. For clarity, we leave most zero cells blank. At the s^1 row there is a negative coefficient; thus, there are two sign changes. The system is unstable, since it has two right–half-plane poles and two left–half-plane poles. The system cannot have $j\omega$ poles since a row of zeros did not appear in the Routh table.

TABLE 6.10 Routh table for Example 6.6

s^4	1	11	200
s^3	~~6~~ 1	~~6~~ 1	
s^2	~~10~~ 1	~~200~~ 20	
s^1	−19		
s^0	20		

Example 6.7

Routh-Hurwitz with Zero in First Column

PROBLEM: Find the number of poles in the left half-plane, the right half-plane, and on the $j\omega$-axis for the system of Figure 6.7.

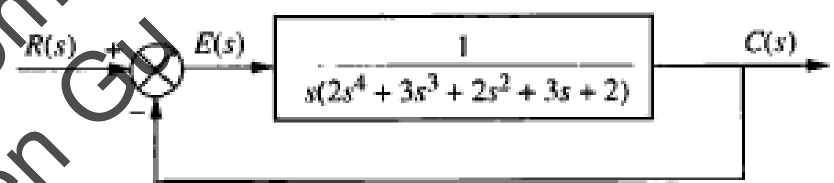


FIGURE 6.7 Feedback control system for Example 6.7

SOLUTION: The closed-loop transfer function is

$$T(s) = \frac{1}{2s^5 + 3s^4 + 2s^3 + 3s^2 + 2s + 1} \tag{6.15}$$

Form the Routh table shown as Table 6.11, using the denominator of Eq. (6.15). A zero appears in the first column of the s^3 row. Since the entire row is not zero, simply replace the zero with a small quantity, ϵ, and continue the table. Permitting ϵ to be a small, positive quantity, we find that the first term of the s^2 row is negative. Thus, there are two sign changes, and the system is unstable, with two poles in the right half-plane. The remaining poles are in the left half-plane.

TABLE 6.11 Routh table for Example 6.7

s^5		2	2	2
s^4		3	3	1
s^3	~~0~~	ϵ	$\frac{4}{3}$	
s^2		$\frac{3\epsilon - 4}{\epsilon}$	1	
s^1		$\frac{12\epsilon - 16 - 3\epsilon^2}{9\epsilon - 12}$		
s^0		1		

Example 6.8

Routh-Hurwitz with Row of Zeros

PROBLEM: Find the number of poles in the left half-plane, the right half-plane, and on the $j\omega$-axis for the system of Figure 6.8. Draw conclusions about the stability of the closed-loop system.

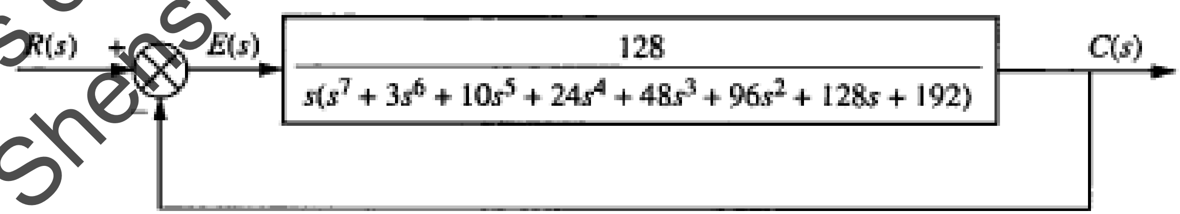


FIGURE 6.8 Feedback control system for Example 6.8

TryIt 6.2

Use MATLAB, The Control System Toolbox, and the following statements to find the closed-loop transfer function, $T(s)$, for Figure 6.8 and the closed-loop poles.

```
numg=128;
deng=[1 3 10 24 ...
 48 96 128 192 0];
G=tf(numg,deng);
T=feedback(G,1)
poles=pole(T)
```

SOLUTION: The closed-loop transfer function for the system of Figure 6.8 is

$$T(s) = \frac{128}{s^8 + 3s^7 + 10s^6 + 24s^5 + 48s^4 + 96s^3 + 128s^2 + 192s + 128} \quad (6.17)$$

Using the denominator, form the Routh table shown as Table 6.13. A row of zeros appears in the s^5 row. Thus, the closed-loop transfer function denominator must have an even polynomial as a factor. Return to the s^6 row and form the even polynomial:

$$P(s) = s^6 + 8s^4 + 32s^2 + 64 \quad (6.18)$$

TABLE 6.13 Routh table for Example 6.8

s					
s^8	1	10	48	128	128
s^7	~~3~~ 1	~~24~~ 8	~~96~~ 32	~~192~~ 64	
s^6	~~3~~ 1	~~16~~ 8	~~64~~ 32	~~128~~ 64	
s^5	~~0~~ ~~6~~ 3	~~0~~ ~~32~~ 16	~~0~~ ~~64~~ 32	~~0~~ ~~0~~ 0	
s^4	~~$\frac{8}{3}$~~ 1	~~$\frac{64}{3}$~~ 8	~~64~~ 24		
s^3	~~-8~~ -1	~~-40~~ -5			
s^2	~~3~~ 1	~~24~~ 8			
s^1	3				
s^0	8				

Differentiate this polynomial with respect to s to form the coefficients that will replace the row of zeros:

$$\frac{dP(s)}{ds} = 6s^5 + 32s^3 + 64s + 0 \tag{6.19}$$

Replace the row of zeros at the s^5 row by the coefficients of Eq. (6.19) and multiply through by 1/2 for convenience. Then complete the table.

We note that there are two sign changes from the even polynomial at the s^6 row down to the end of the table. Hence, the even polynomial has two right–half-

TABLE 6.14 Summary of pole locations for Example 6.8

	Polynomial		
Location	**Even (sixth-order)**	**Other (second-order)**	**Total (eighth-order)**
Right half-plane	2	0	2
Left half-plane	2	2	4
$j\omega$	2	0	2

plane poles. Because of the symmetry about the origin, the even polynomial must have an equal number of left–half-plane poles. Therefore, the even polynomial has two left–half-plane poles. Since the even polynomial is of sixth order, the two remaining poles must be on the $j\omega$-axis.

There are no sign changes from the beginning of the table down to the even polynomial at the s^6 row. Therefore, the rest of the polynomial has no right–half-plane poles. The results are summarized in Table 6.14. The system has two poles in the right half-plane, four poles in the left half-plane, and two poles on the $j\omega$-axis, which are of unit multiplicity. The closed-loop system is unstable because of the right–half-plane poles.

Example 6.9

Stability Design via Routh-Hurwitz

PROBLEM: Find the range of gain, K, for the system of Figure 6.10 that will cause the system to be stable, unstable, and marginally stable. Assume $K > 0$.

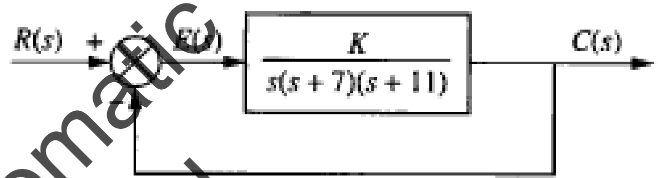


FIGURE 6.10 Feedback control system for Example 6.9

SOLUTION: First find the closed-loop transfer function as

$$T(s) = \frac{K}{s^3 + 18s^2 + 77s + K} \tag{6.20}$$

Next form the Routh table shown as Table 6.15.

TABLE 6.15 Routh table for Example 6.9

s^3	1	77
s^2	18	K
s^1	$\frac{1386 - K}{18}$	
s^0	K	

Since K is assumed positive, we see that all elements in the first column are always positive except the s^1 row. This entry can be positive, zero, or negative, depending upon the value of K. If $K < 1386$, all terms in the first column will be positive, and since there are no sign changes, the system will have three poles in the left half-plane and be *stable*.

If $K > 1386$, the s^1 term in the first column is negative. There are two sign changes, indicating that the system has two right–half-plane poles and one left–half-plane pole, which makes the system *unstable*.

If $K = 1386$, we have an entire row of zeros, which could signify $j\omega$ poles. Returning to the s^2 row and replacing K with 1386, we form the even polynomial

$$P(s) = 18s^2 + 1386 \tag{6.21}$$

Differentiating with respect to s, we have

$$\frac{dP(s)}{ds} = 36s + 0 \tag{6.22}$$

Replacing the row of zeros with the coefficients of Eq. (6.22), we obtain the Routh-Hurwitz table shown as Table 6.16 for the case of $K = 1386$.

TABLE 6.16 Routh table for Example 6.9 with $K = 1386$

s^3	1	77
s^2	18	1386
s^1	~~0~~ 36	
s^0	1386	

Since there are no sign changes from the even polynomial (s^2 row) down to the bottom of the table, the even polynomial has its two roots on the $j\omega$-axis of unit multiplicity. Since there are no sign changes above the even polynomial, the remaining root is in the left half-plane. Therefore the system is *marginally stable*.

Skill-Assessment Exercise 6.3

WileyPLUS
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Control Solutions

PROBLEM: For a unity feedback system with the forward transfer function

$$G(s) = \frac{K(s+20)}{s(s+2)(s+3)}$$

find the range of K to make the system stable.

ANSWER: $0 < K < 2$

The complete solution is at www.wiley.com/college/nise.

6.3

Since $G(s) = \dfrac{K(s+20)}{s(s+2)(s+3)}$, $T(s) = \dfrac{G(s)}{1+G(s)} = \dfrac{K(s+20)}{s^3+5s^2+(6+K)s+20K}$

Form the Routh table.

s^3	1	$(6+K)$
s^2	5	$20K$
s^1	$\dfrac{30-15K}{5}$	
s^0	$20K$	

From the s^1 row, $K < 2$. From the s^0 row, $K > 0$. Thus, for stability, $0 < K < 2$.

Summary

- Stability definition for linear systems from the viewpoint of natural response:
  - Stable: Natural response decays to zero as time approaches infinity;
  - Unstable: Natural response increases without bound;
  - Marginally stable: Natural response neither decay nor grow without bound but oscillate.
- Stability definition for linear systems from the viewpoint of total response:
  - Stable: If every bounded input yields a bounded output (BIBO);
  - Unstable: If any bounded input yields an unbounded output.

Summary (Cont'd)

- Stability for linear time-invariant systems can be determined from the location of the closed-loop poles:
  - If the poles are only in the left half-plane, the system is stable;
  - If any poles are in the right half-plane, the system is unstable;
  - If the poles are on the imaginary axis and in the left half-plane, the system is marginally stable as long as the poles on the imaginary axis are of unit multiplicity; it is unstable if there are any multiple imaginary poles.
- The ***Routh-Hurwitz criterion*** lets us find how many poles are in each of the sections of the s-plane without giving us the coordinates of the poles.
- Just knowing that there are poles in the right half-plane is enough to determine that a system is unstable.