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Principles of Automatic Control (1)

自动控制原理1

Topic 4

Reduction of Multiple Subsystems

(Chapter 5 in text book)

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Review of Time Response

- Inverse transformation is required to get back to time domain;
- Poles and zeros of transfer function;
- Natural response is due to poles;
- Amplitude is affected by both poles and zeros;
- Pole on real axis causes exponential response;
- First order system and time constant (one possible response);
- Second order system – Four possible responses combination of poles;
- Second order system: Natural frequency ω_n and damping ration ζ;
- Pole movement vs. Step response
- Time response for complex system with several subsystems?

New terminologies in this topic

- Block diagram 方框图
- Signal-flow diagram 信号流图
- Mason’s rule 梅森公式
- Summing junction 比较点
- Pickoff point 引出点
- Cascade 串联
- Parallel 并联
- Feedback 反馈
- Open-loop transfer function 开环传递函数
- Gain 增益
- Node 节点

- Loop 环路
- Loop gain 环路增益
- Path 通路
- Forward-path gain 前向通路增益
- Nontouching loops 非接触环路
- Nontouching-loop gain 非接触环路增益
- Traverse 穿过

Learning Outcomes for Topic 4

After completing this topic, you will be able to:

- Reduce a block diagram of **MULTIPLE** subsystems to a **SINGLE** block representing the transfer function from input to output;
- Analyze and design transient response for a system consisting of multiple subsystems;
- Convert block diagrams to signal-flow diagrams;
- Find the transfer function of multiple subsystems using Mason's rule.

Outline

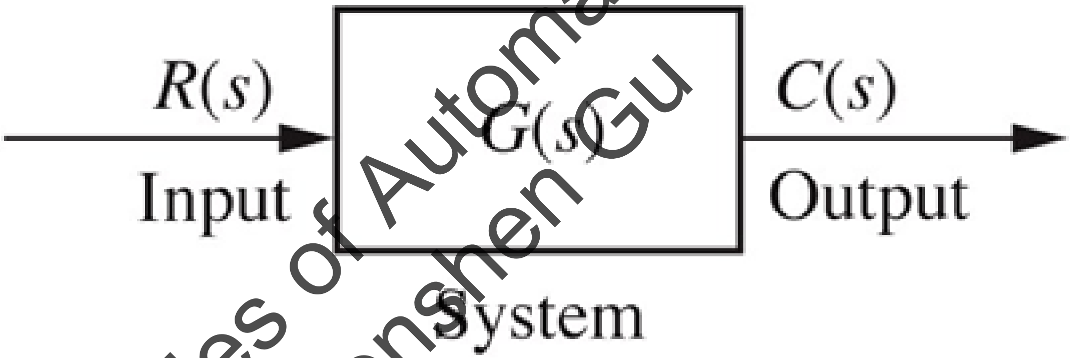
- Brief Introduction
- Block Diagrams
- Analysis and Design of Feedback Systems
- Signal-Flow Graphs
- Mason's Rule
- Summary

Brief Introduction

- So far we have been dealing with *G(s)* the transfer function of the plant, without any consideration to the various subsystems. In complex systems, there will be several subsystems and also can have multiple feedback loops etc.
- Since the response of a single transfer function can be calculated, we want to represent multiple subsystems as a single transfer function.
- We can then apply the analytical techniques of the previous topics and obtain transient response information about the entire system.
- Multiple subsystems are represented in two ways: as block diagrams and as signal-flow graphs.
- We will develop techniques to reduce each representation to a single transfer function. Block diagram algebra will be used to reduce block diagrams and Mason's rule to reduce signal-flow graphs.

Block Diagrams

- We already know, a subsystem is represented as a block with an input, an output, and a transfer function.



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- Many systems are composed of multiple subsystems, for example the space shuttle.

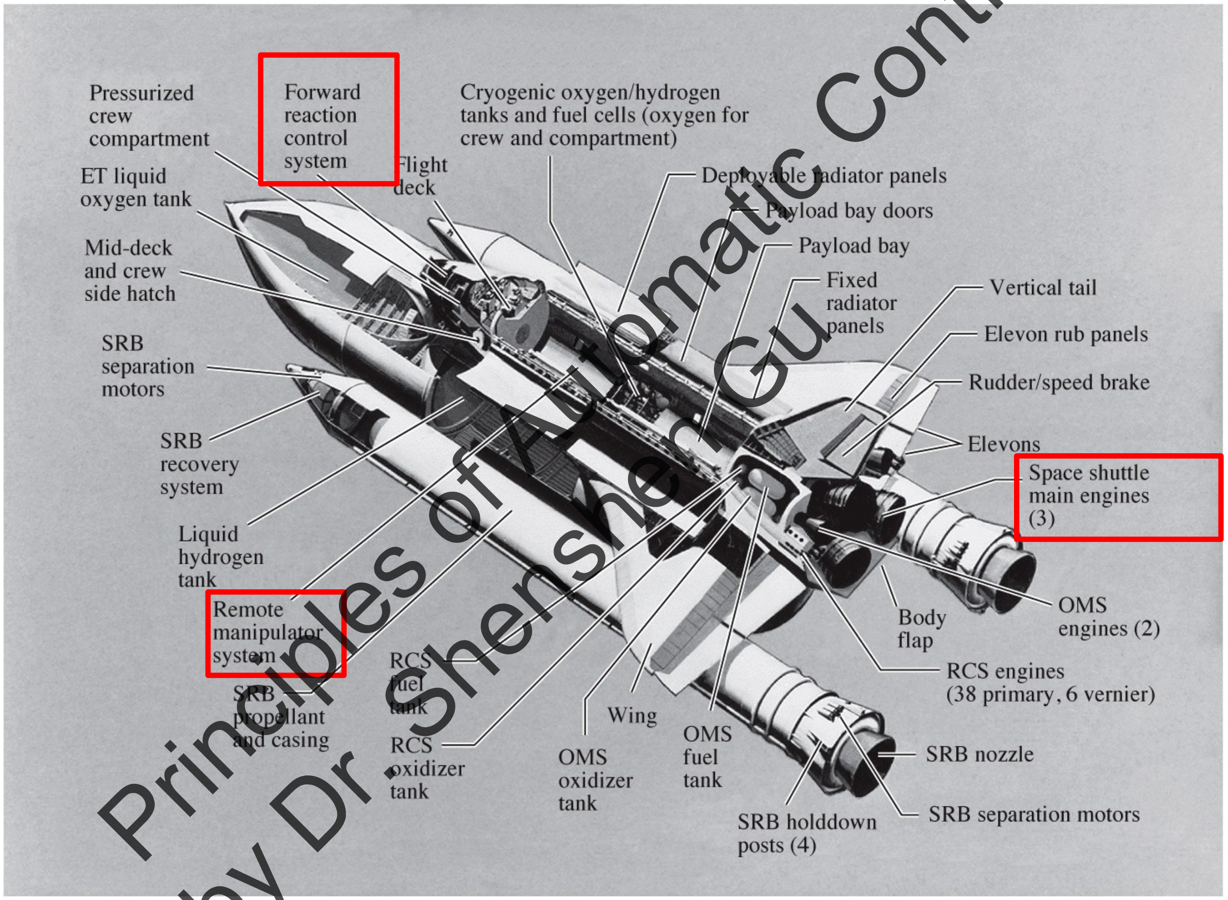
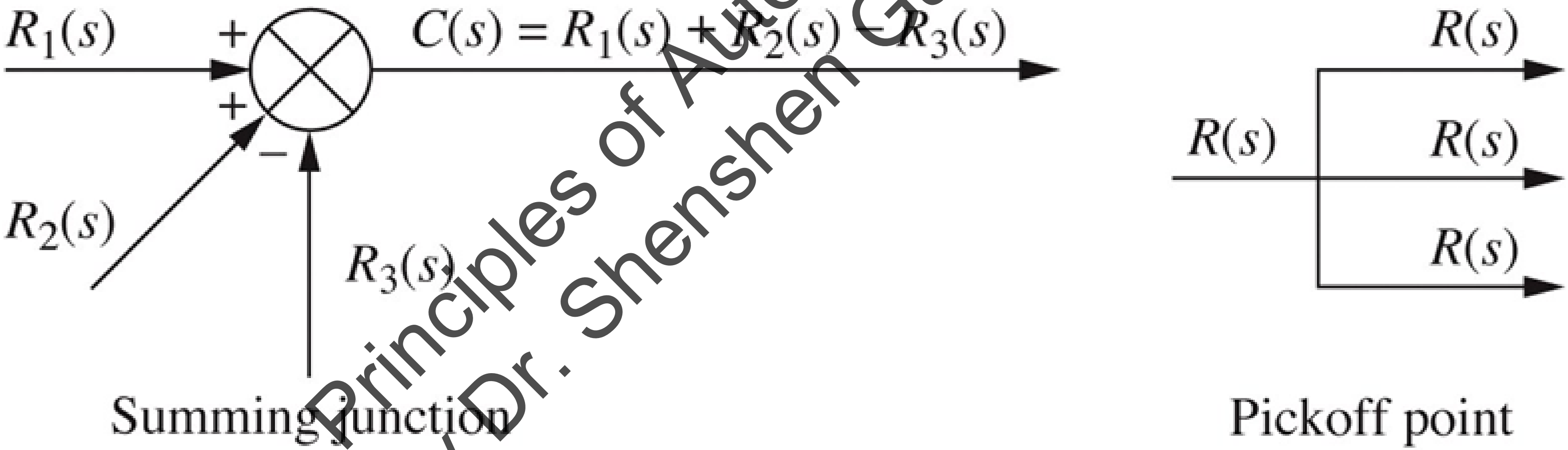
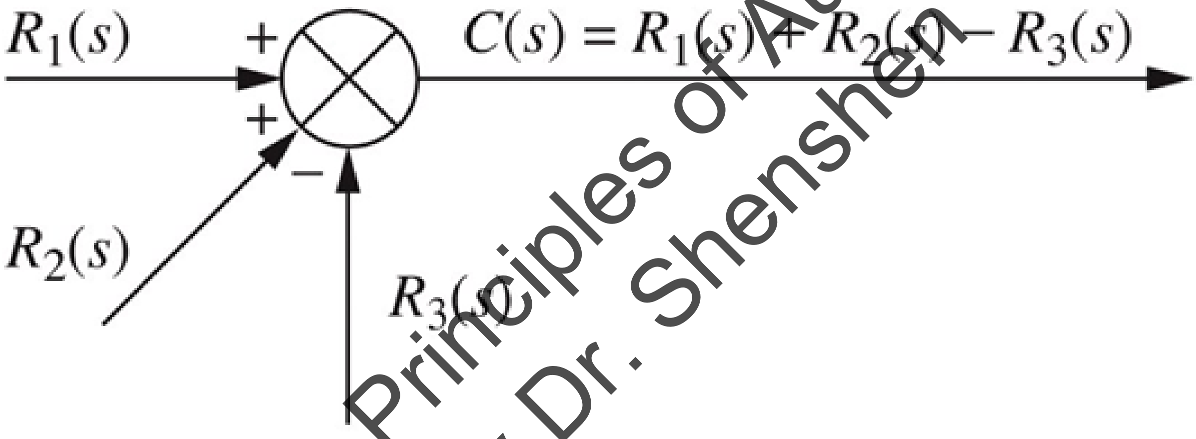


Figure 5.1

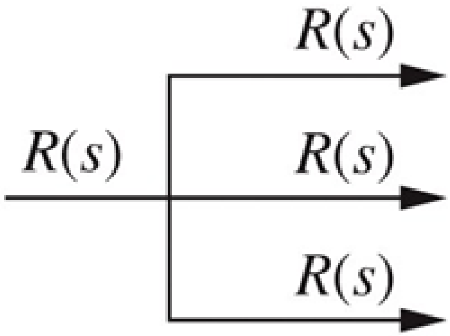
- When multiple subsystems are interconnected, a few more schematic elements must be added to the block diagram.
- These new elements are ***summing junctions*** and ***pickoff points***.



- The characteristic of the summing junction is that the output signal, *C(s)*, is the ***algebraic sum*** of the input signals, *$R_1(s)$*, *$R_2(s)$ and $R_3(s)$.*
- A pickoff point distributes the input signal, *R(s)*, ***undiminished***, *to several output points.*



Summing junction

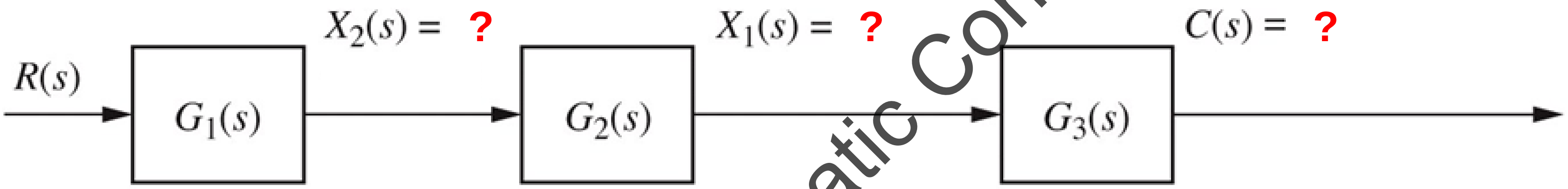


Pickoff point

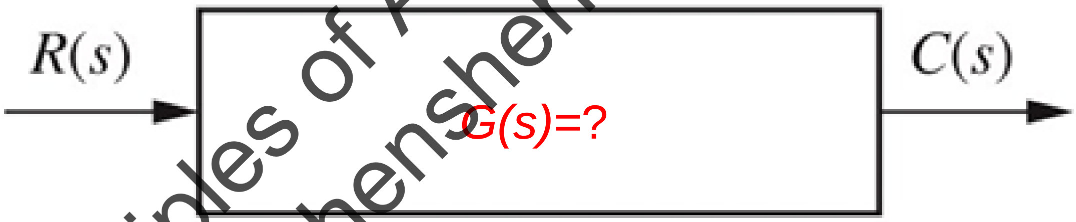
Before we dealing with the complicated systems

- Examine some common topologies for interconnecting subsystems;
  - Cascade form;
  - Parallel form;
  - Feedback form;
- Derive the single transfer function representation for these forms;
- These common topologies will form the basis for reducing more complicated systems to a single block.

Cascade Form

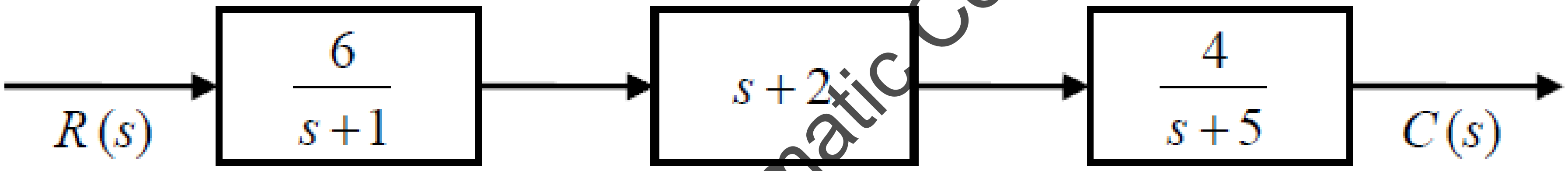


(a)



(b)

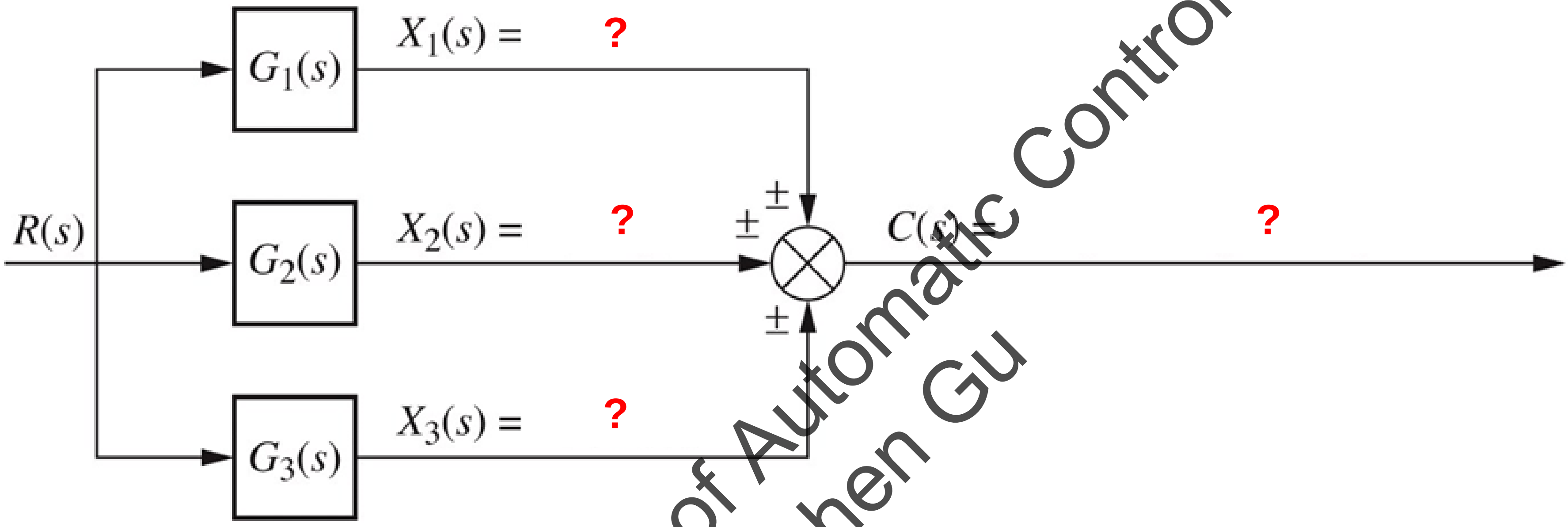
- Find the single transfer function for:

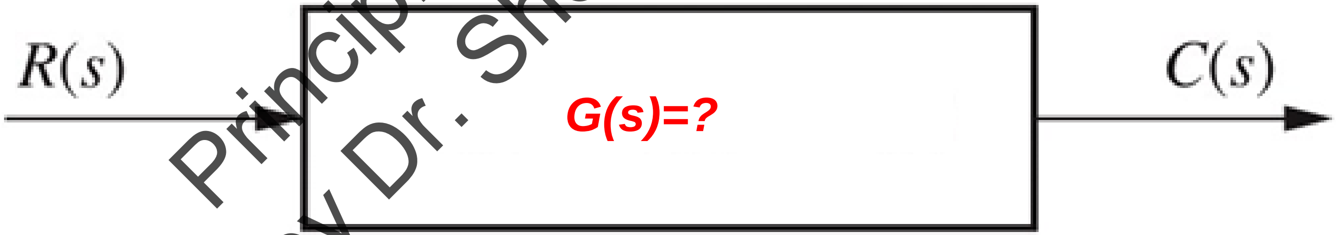


$$G(s)=\frac{6}{s+1}(s+2)\frac{4}{s+5}$$

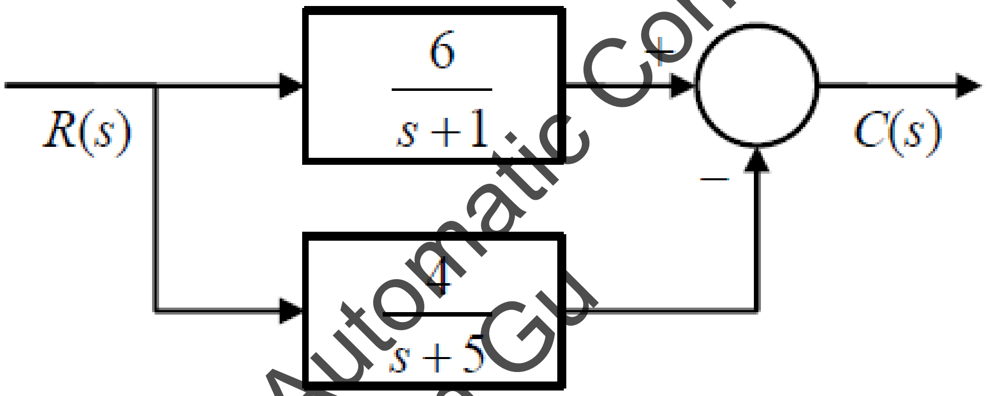
$$=\frac{24(s+2)}{(s+1)(s+5)}$$

Parallel Form





- Find the single transfer function for:

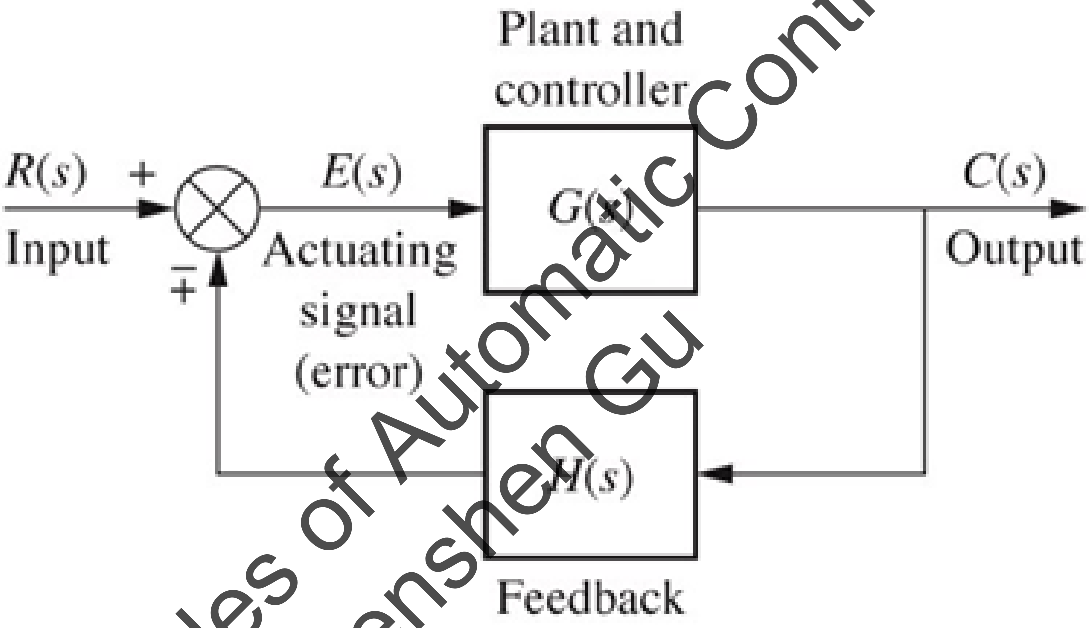


$$G(s)=\frac{6}{s+1}-\frac{4}{s+5}$$

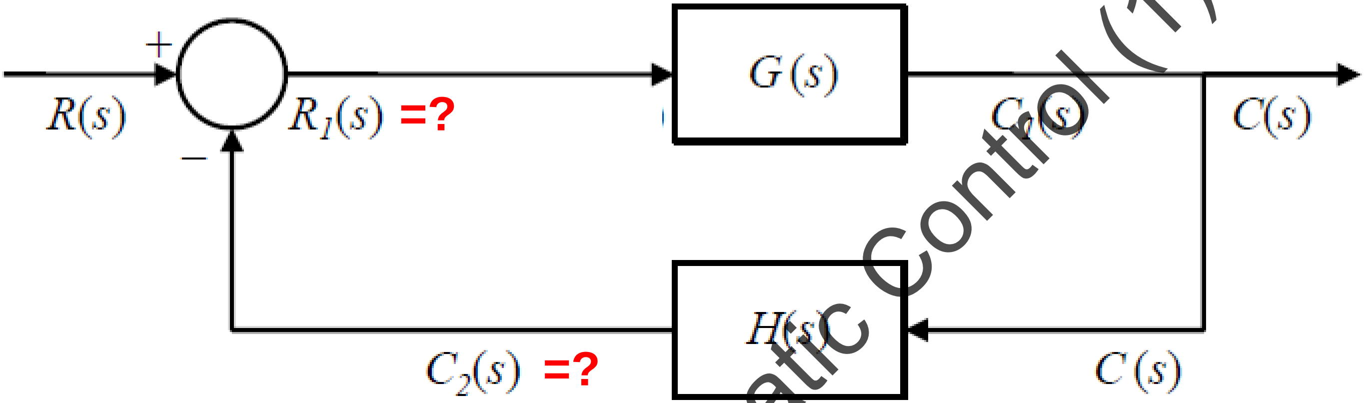
$$=\frac{6(s+5)-4(s+1)}{(s+1)(s+5)}$$

$$=\frac{2s+26}{(s+1)(s+5)}$$

Feedback Form



Negative Feedback Systems



$$= H(s)C(s)$$

$$C(s) = R_1(s)G(s)$$

$$= \left(R(s) - C_2(s)\right)G(s)$$

$$= \left(R(s) - H(s)C(s)\right)G(s)$$

$$= R(s)G(s) - C(s)H(s)G(s)$$

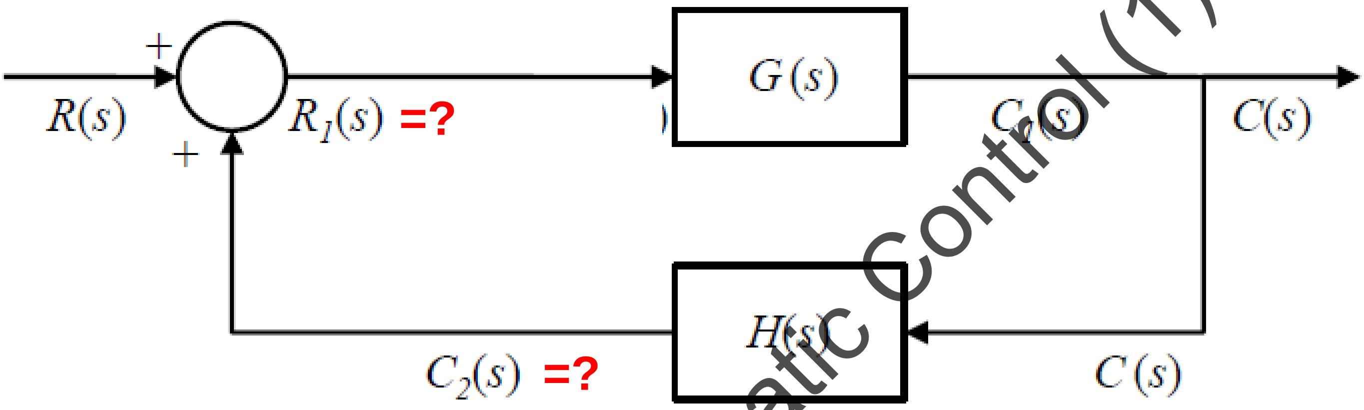
$$R(s)G(s) = C(s) + C(s)H(s)G(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$\boxed{G_e(s) = \frac{G(s)}{1+\boxed{G(s)H(s)}}}$$

Open-loop transfer function

Positive Feedback Systems



$$= H(s)C(s)$$

$$C(s) = R_1(s)G(s)$$

$$= \left(R(s) + C_2(s)\right)G(s)$$

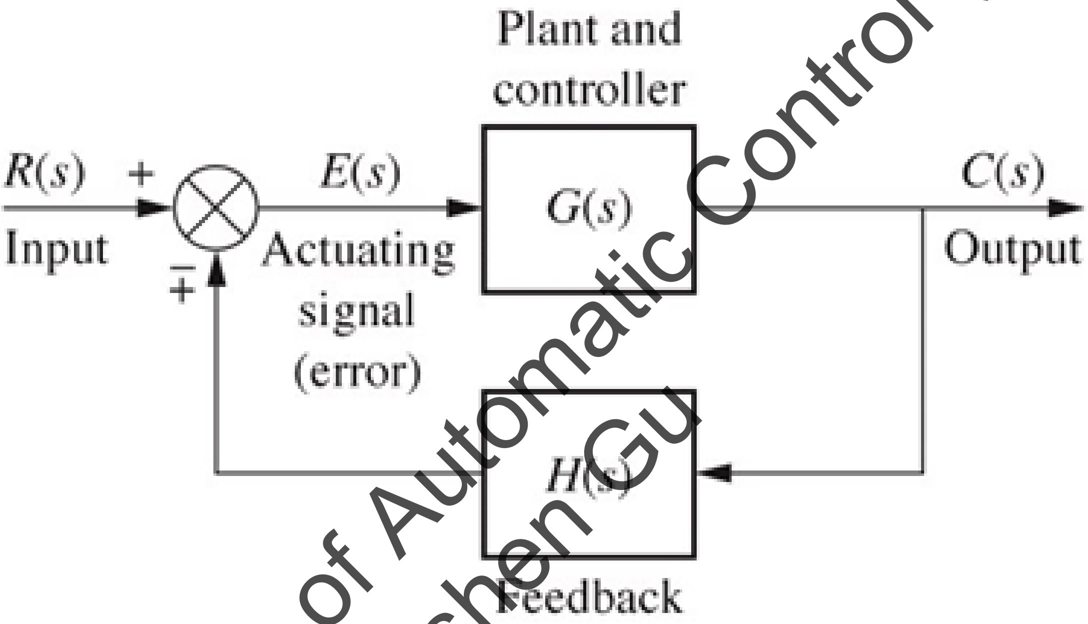
$$= \left(R(s) + H(s)C(s)\right)G(s)$$

$$= R(s)G(s) + C(s)H(s)G(s)$$

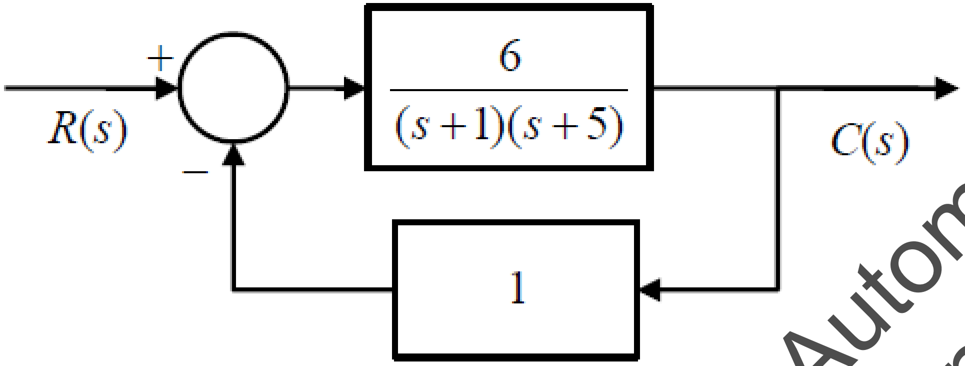
$$R(s)G(s) = C(s) - C(s)H(s)G(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

$$\boxed{G_e(s) = \frac{G(s)}{1 - G(s)H(s)}}$$



$$G_e(s) = \frac{G(s)}{1 \pm G(s)H(s)}$$



$$\boxed{G_e(s)=\frac{G(s)}{1+G(s)H(s)}}$$

$$G(s)=\frac{\dfrac{6}{(s+1)(s+5)}}{1+\dfrac{6}{(s+1)(s+5)}}$$

$$=\frac{6}{(s+1)(s+5)+6}$$

$$=\frac{6}{s^2+6s+11}$$

Reduction Formulae

- Cascade: (Multiplication)

$$G(s)=G_1(s)G_2(s)\cdots G_n(s)$$

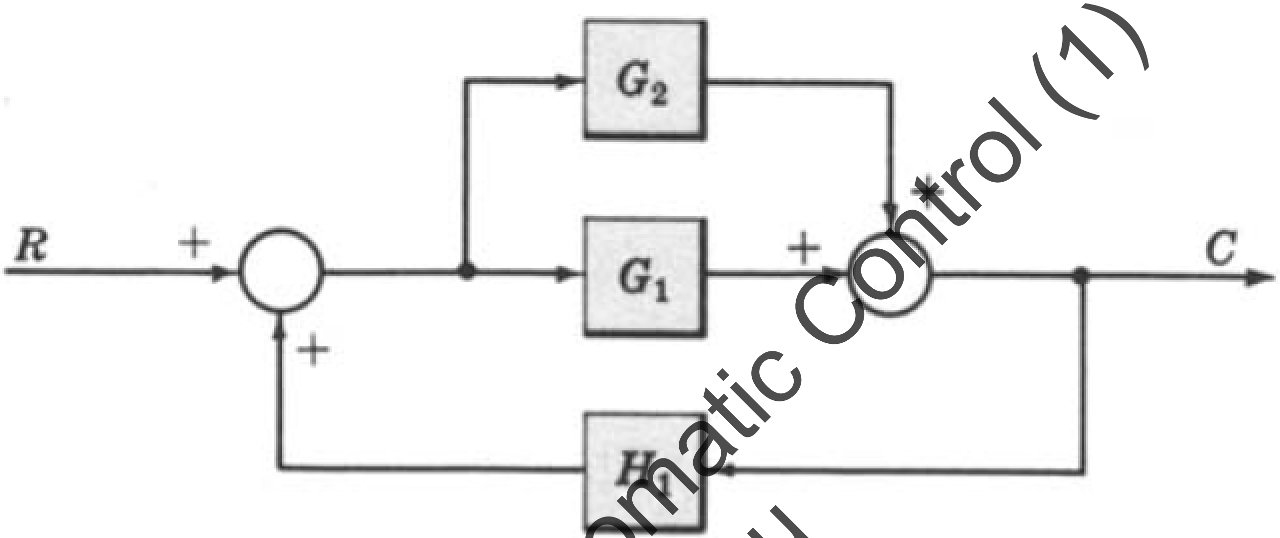
- Parallel: (Addition)

$$G(s)=G_1(s)+G_2(s)+\cdots+G_n(s)$$

- Feedback:

$$G_c(s)=\frac{G(s)}{1\pm G(s)H(s)}$$

Two minutes quiz



(A) $G_e(s)=\dfrac{G_1G_2}{1-H_1(G_1G_2)}$

(B) $G_e(s)=G_1+G_2+H_1$

(C) $G_e(s)=\dfrac{G_1+G_2}{1+H_1(G_1+G_2)}$

[✓ (D)] $G_e(s)=\dfrac{G_1+G_2}{1-H_1(G_1+G_2)}$

Moving Summing Junctions

- It should be noted that familiar forms (cascade, parallel, and feedback) are not always apparent in a block diagram.
- If you move a summing junction after a block, cascade the transfer function.
- If you move before a block, cascade the *inverse transfer function.*

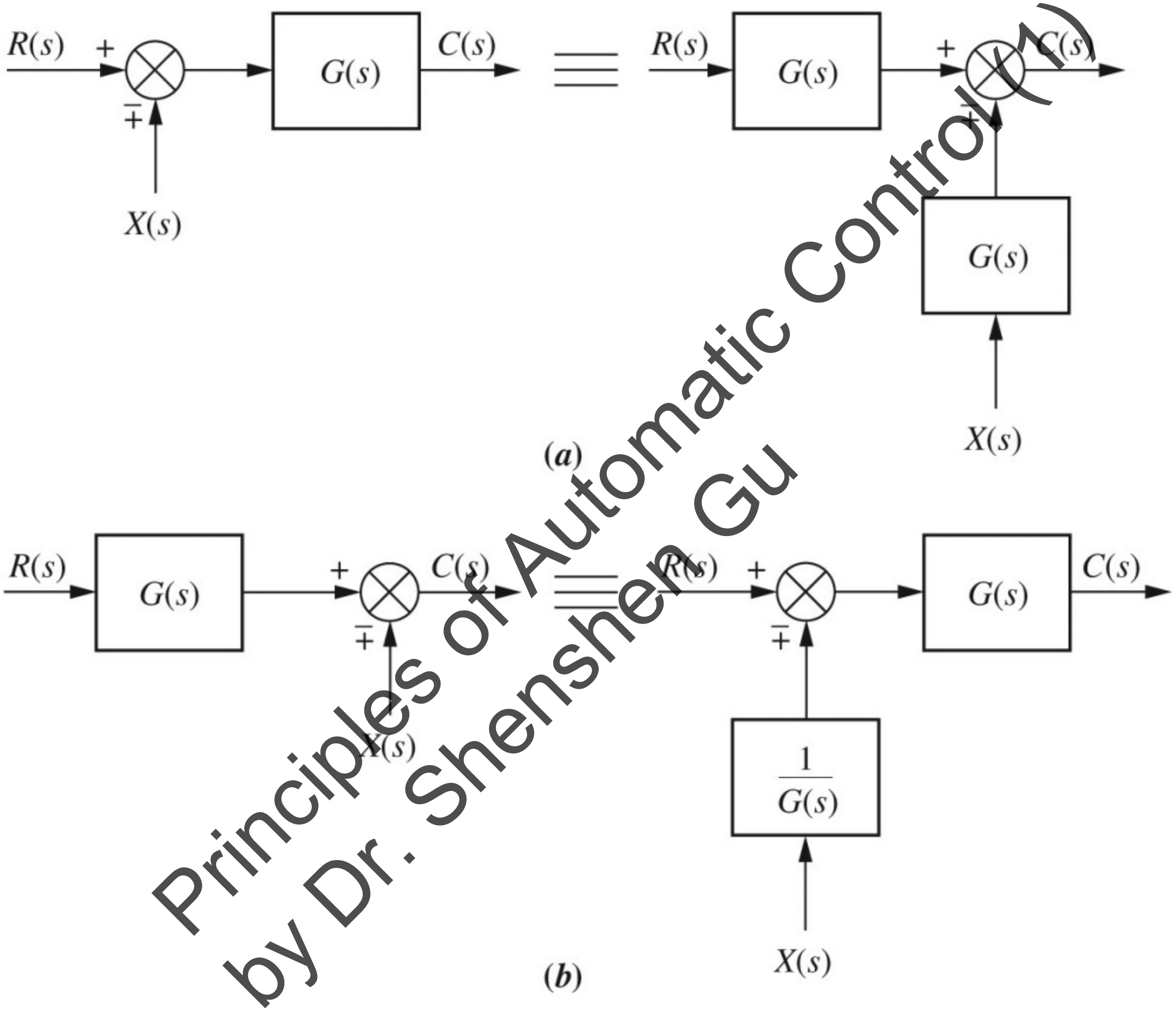
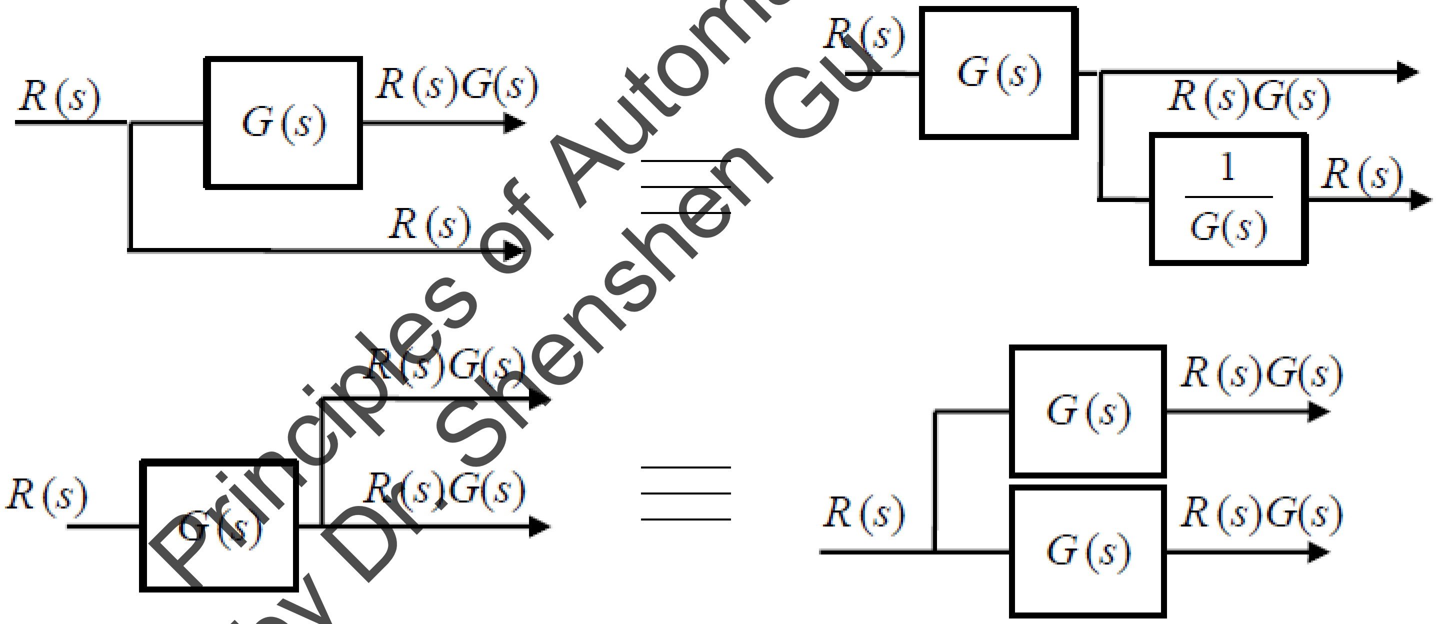


Figure 5.7

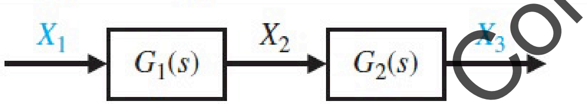
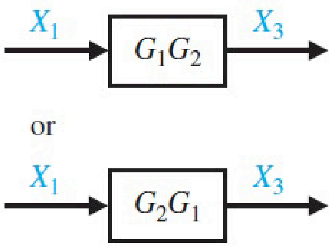
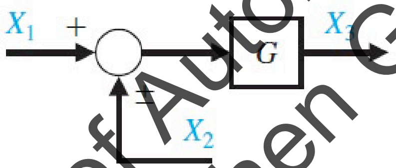
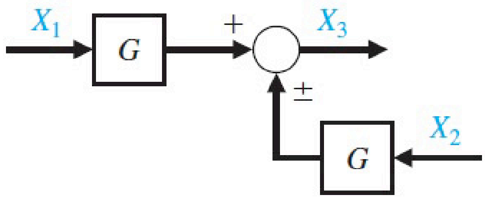
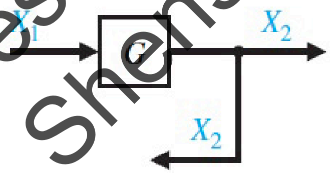
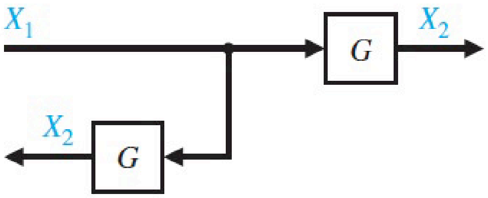
Moving Pick Off Points

- If you move a pick off point after a block, cascade the *inverse transfer function.*
- If you move before a block, cascade the transfer function.

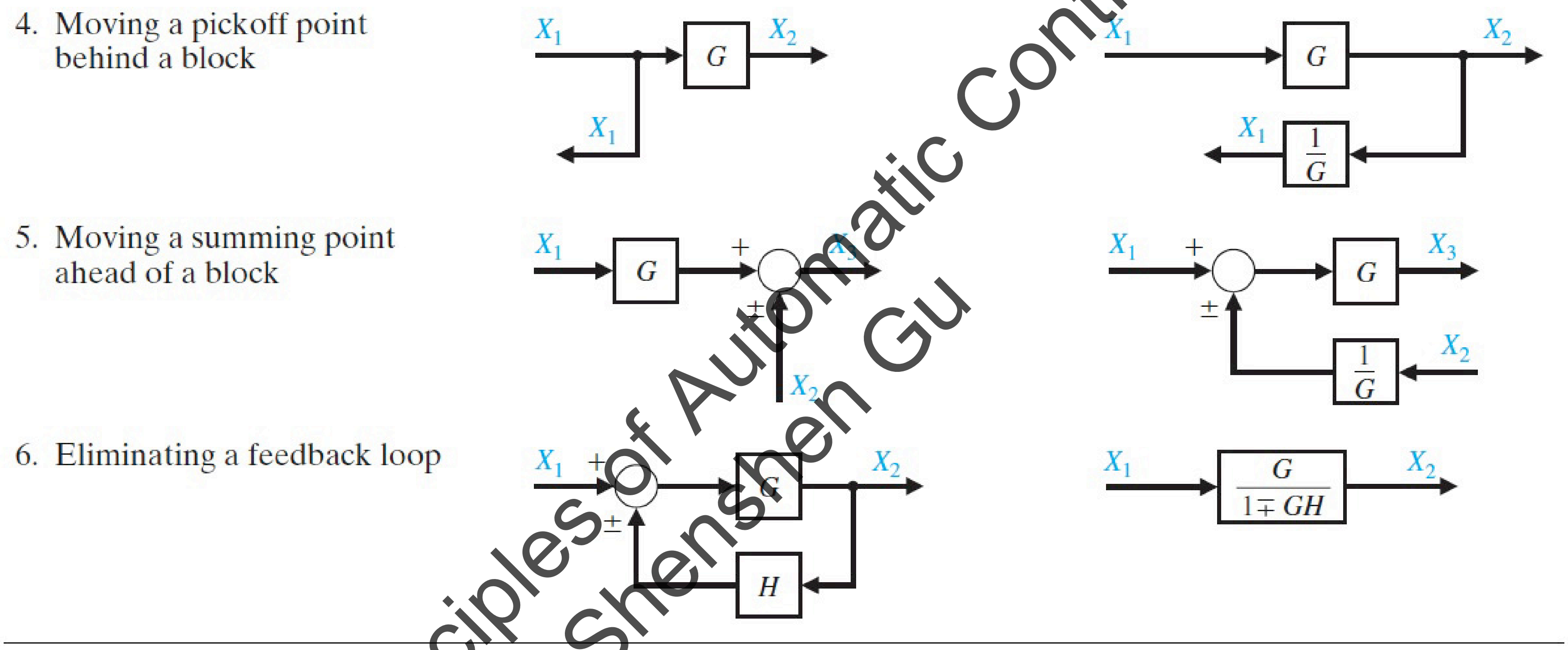


Block Diagram Transformations Summary

Table 2.6 Block Diagram Transformations

Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade		
2. Moving a summing point behind a block		
3. Moving a pickoff point ahead of a block		

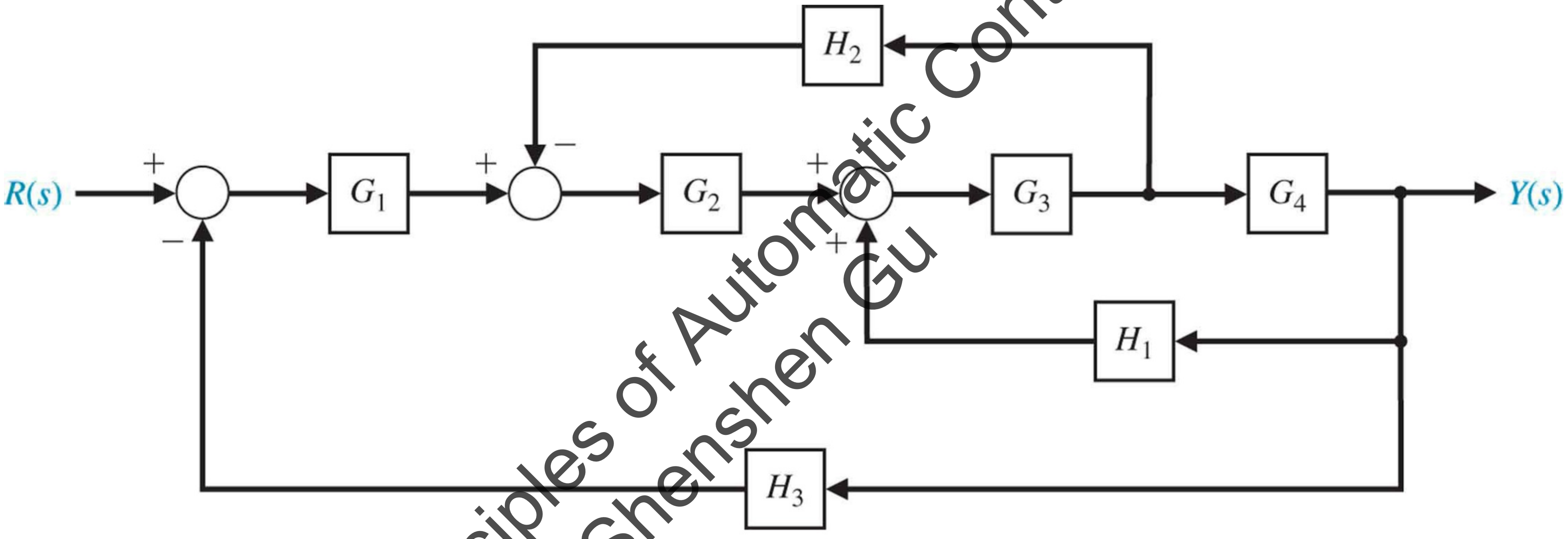
Block Diagram Transformations Summary (Cont'd)



4. Moving a pickoff point behind a block

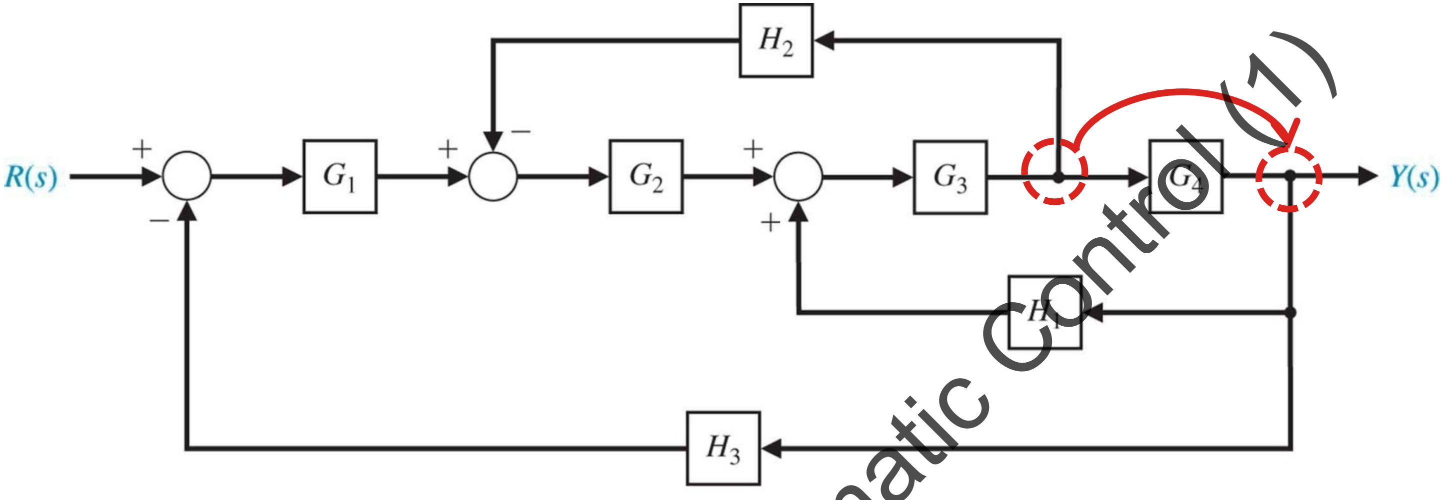
5. Moving a summing point ahead of a block

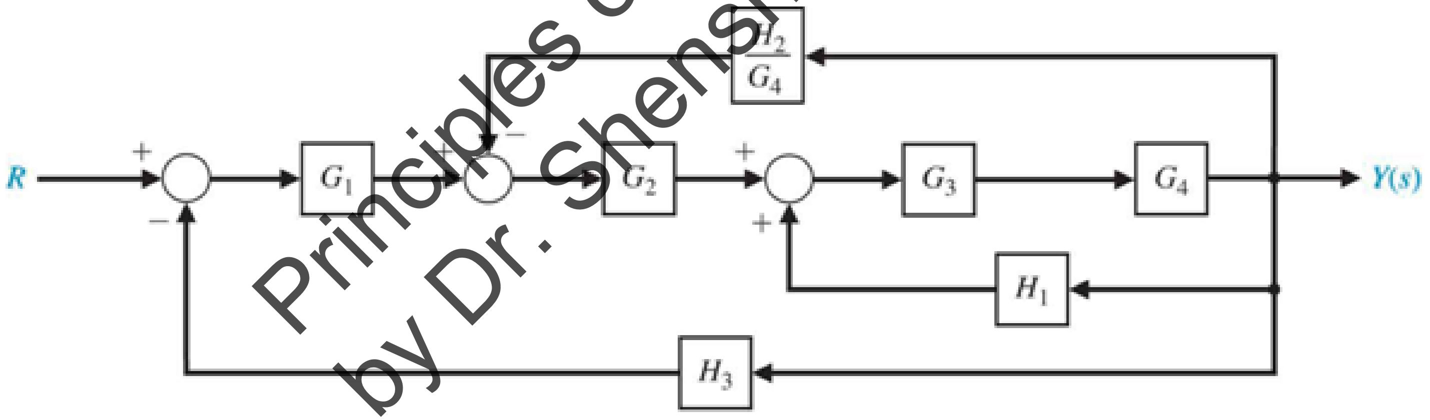
6. Eliminating a feedback loop

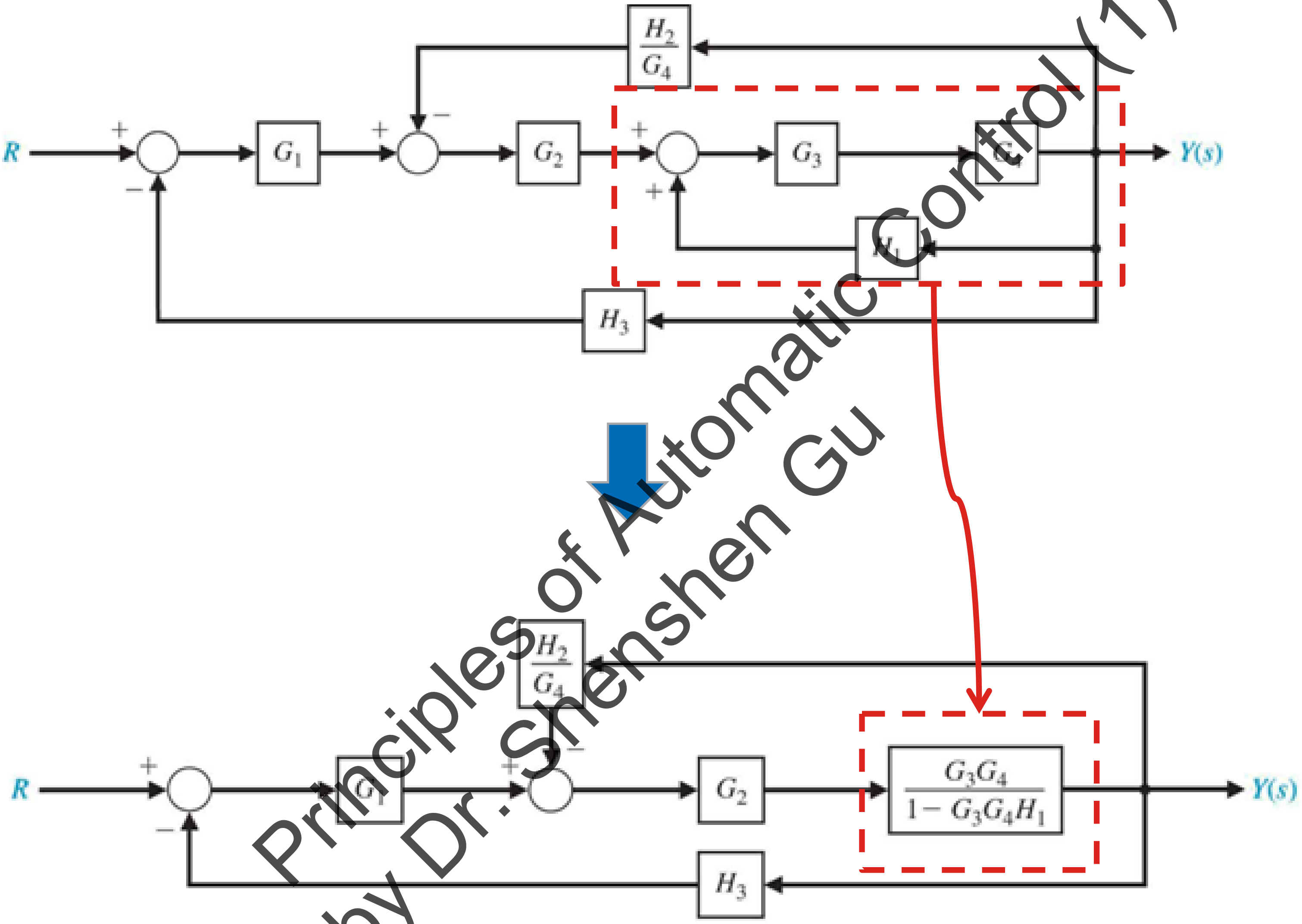


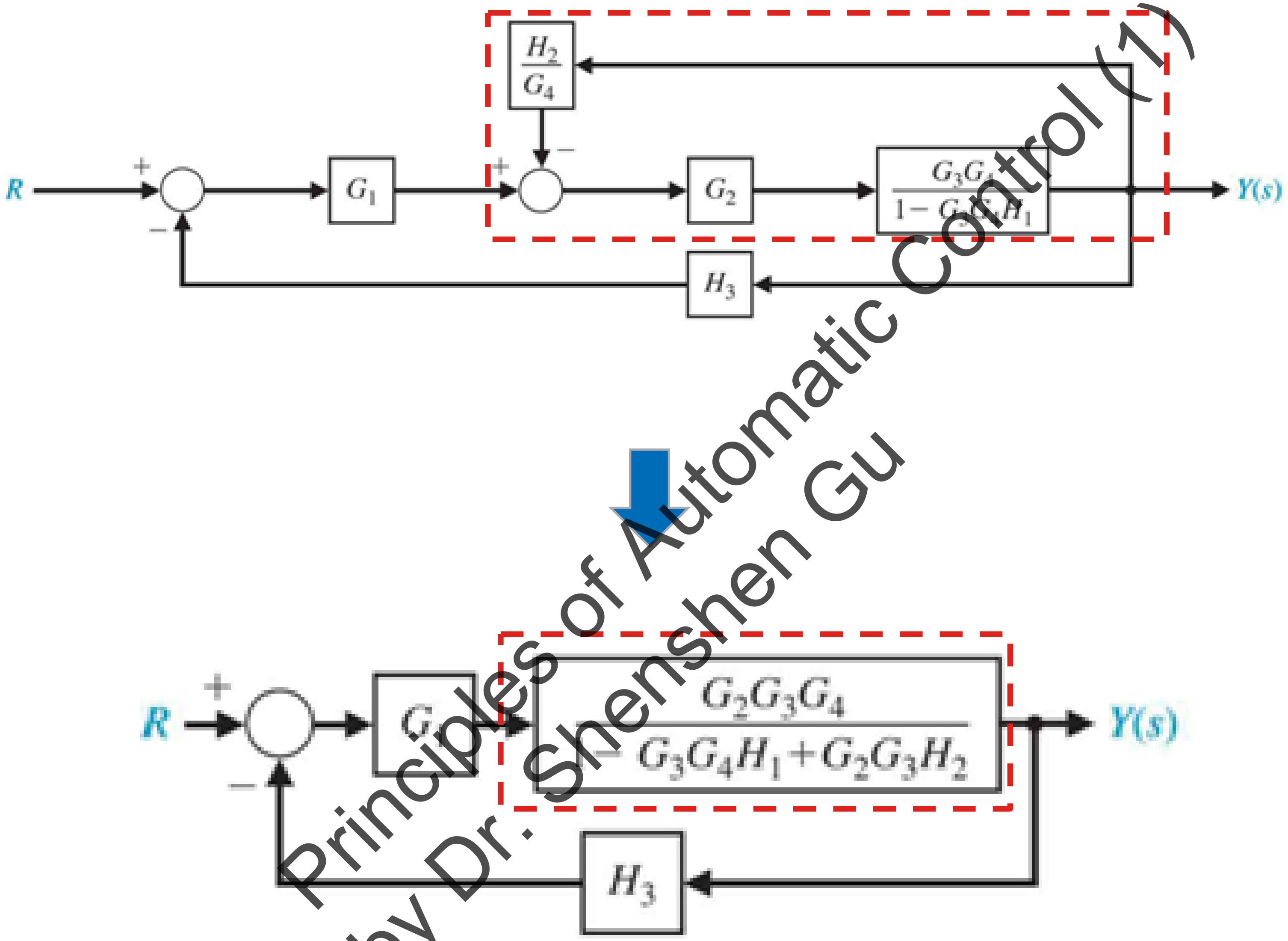
R(s)
+
−
G_1
+
−
H_2
G_2
+
+
G_3
G_4
Y(s)
H_1
H_3

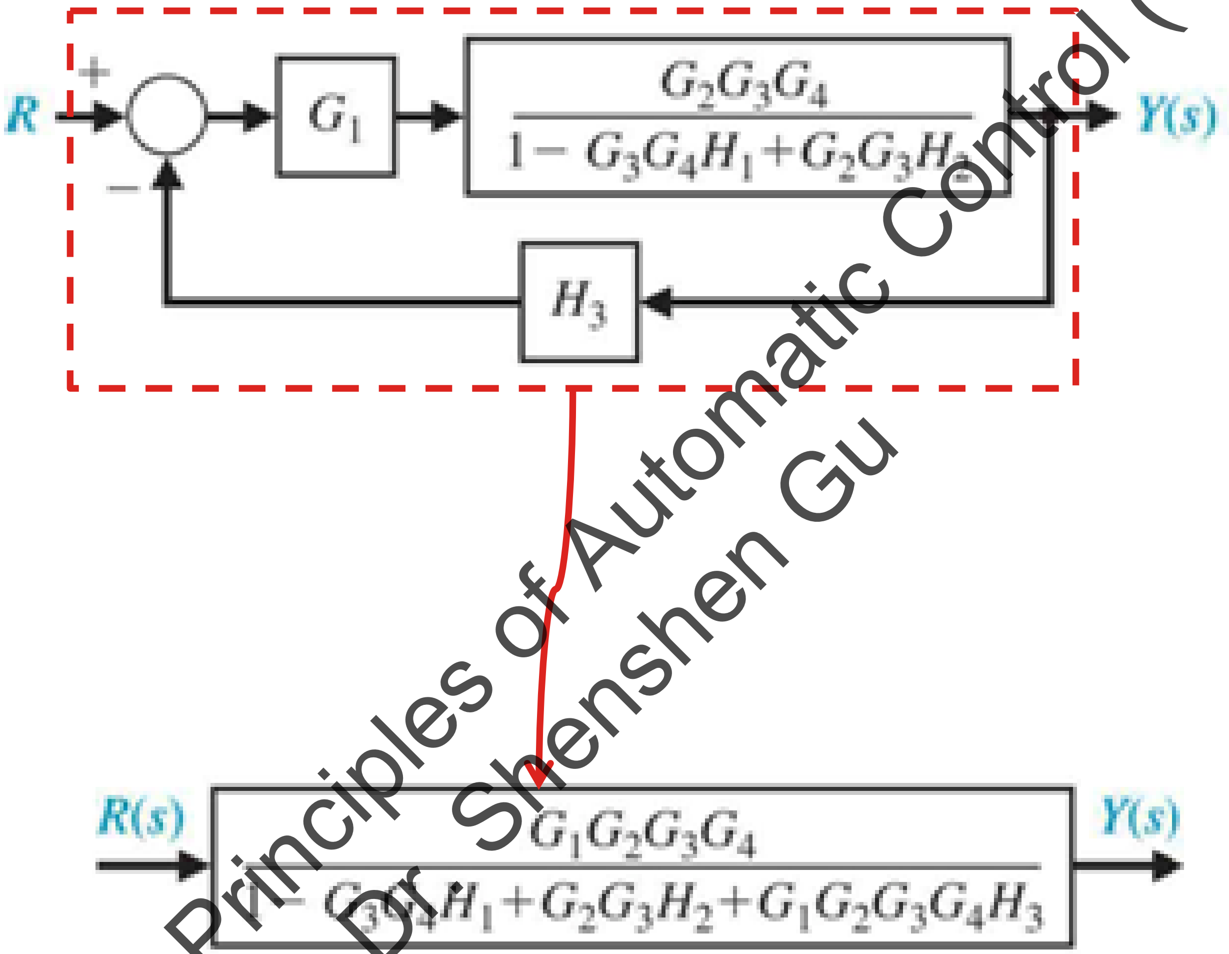
Example

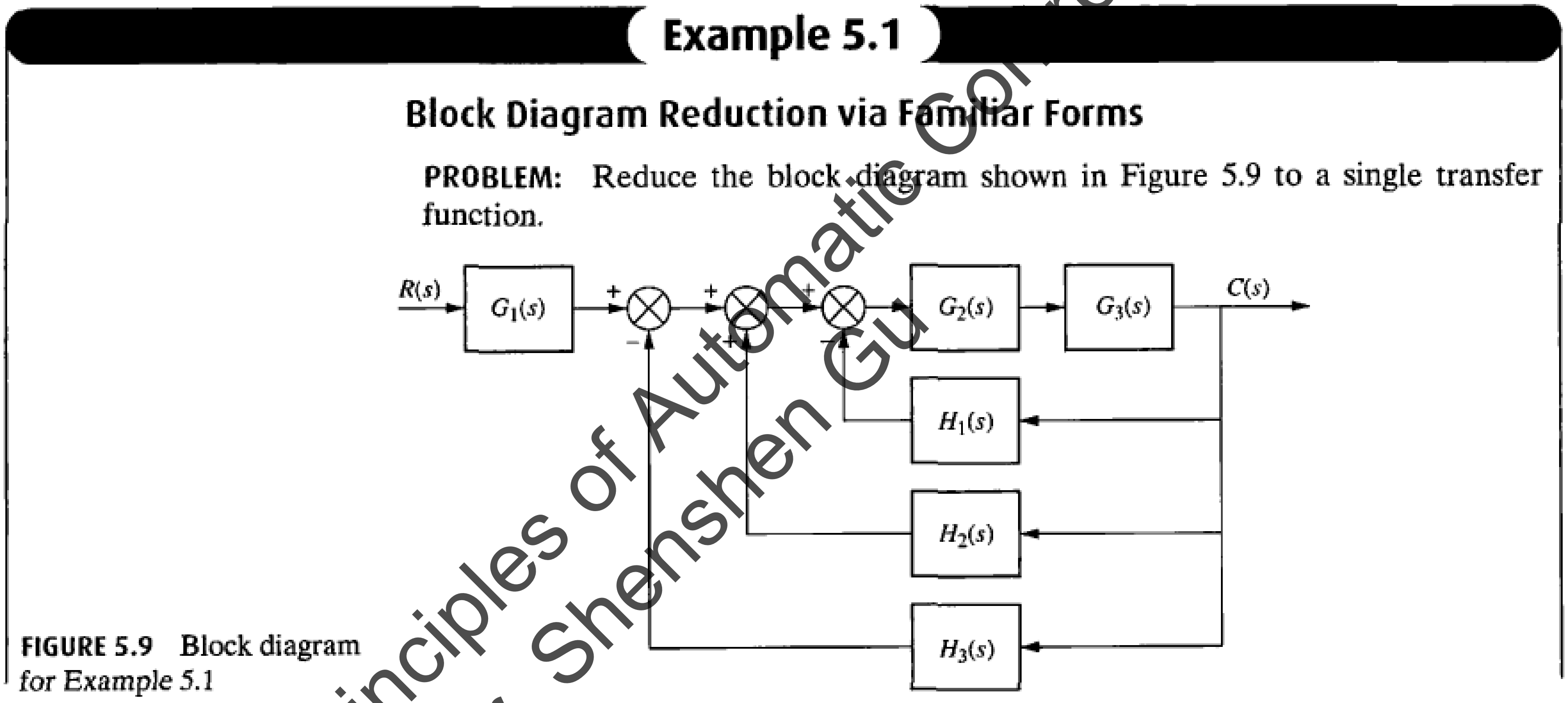





R
+
−
G_1
+
−
$\frac{H_2}{G_4}$
G_2
+
+
G_3
G_4
H_1
H_3
Y(s)
R
+
−
G_1
+
−
$\frac{H_2}{G_4}$
G_2
$\frac{G_3G_4}{1-G_3G_4H_1}$
H_3
Y(s)


R
+
−
G1
H2/G4
+
−
G2
G3G4 / (1 − G3G4H1)
Y(s)
H3
R
+
−
G1
G2G3G4 / (1 − G3G4H1 + G2G3H2)
Y(s)
H3


R
+
−
G_1
$\frac{G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2}$
Y(s)
H_3
R(s)
$\frac{G_1G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4H_3}$
Y(s)

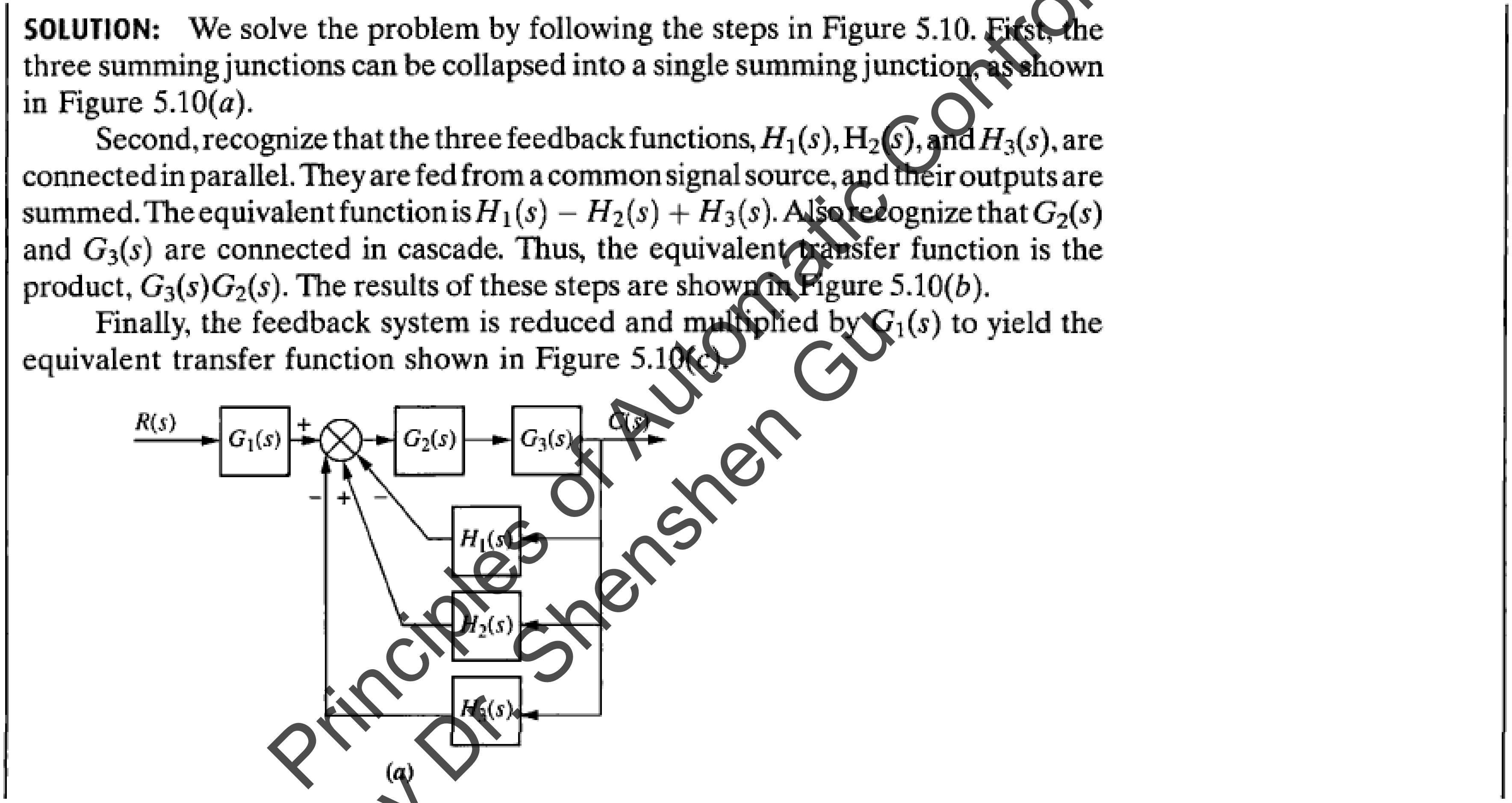


Example 5.1

Block Diagram Reduction via Familiar Forms

PROBLEM: Reduce the block diagram shown in Figure 5.9 to a single transfer function.

FIGURE 5.9 Block diagram for Example 5.1



SOLUTION: We solve the problem by following the steps in Figure 5.10. First, the three summing junctions can be collapsed into a single summing junction, as shown in Figure 5.10(*a*).

Second, recognize that the three feedback functions, $H_1(s)$, $H_2(s)$, and $H_3(s)$, are connected in parallel. They are fed from a common signal source, and their outputs are summed. The equivalent function is $H_1(s) - H_2(s) + H_3(s)$. Also recognize that $G_2(s)$ and $G_3(s)$ are connected in cascade. Thus, the equivalent transfer function is the product, $G_3(s)G_2(s)$. The results of these steps are shown in Figure 5.10(*b*).

Finally, the feedback system is reduced and multiplied by $G_1(s)$ to yield the equivalent transfer function shown in Figure 5.10(*c*).

(*a*)

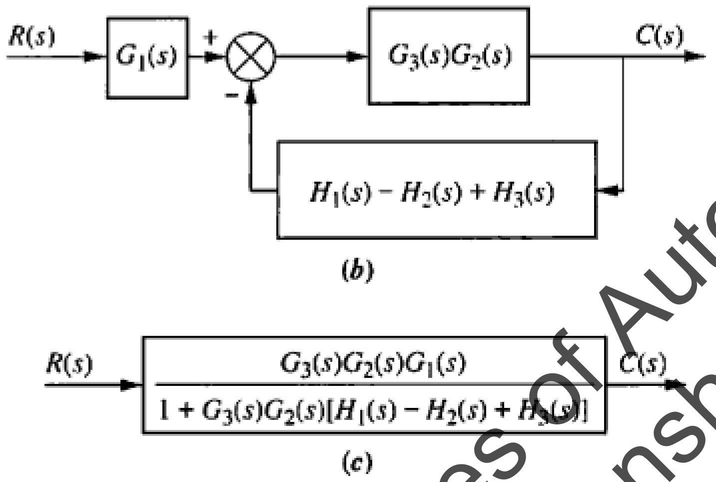


FIGURE 5.10 Steps in solving Example 5.1: **a.** Collapse summing junctions; **b.** form equivalent cascaded system in the forward path and equivalent parallel system in the feedback path; **c.** form equivalent feedback system and multiply by cascaded $G_1(s)$

Example 5.2

Block Diagram Reduction by Moving Blocks

PROBLEM: Reduce the system shown in Figure 5.11 to a single transfer function.

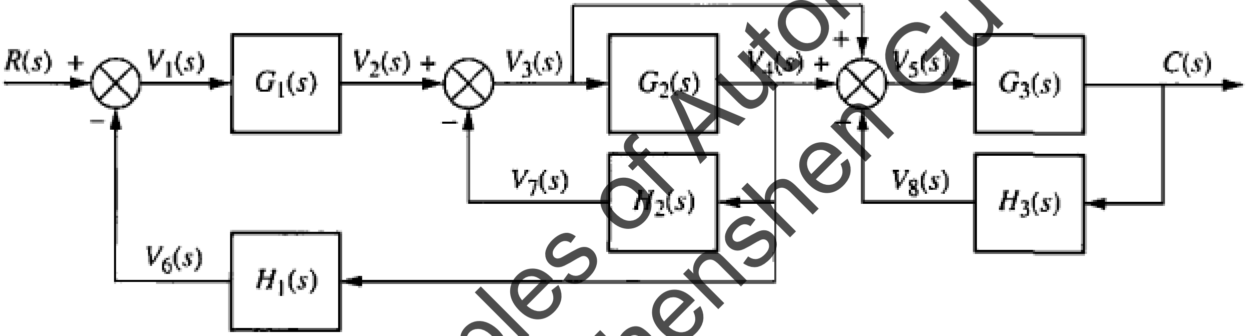
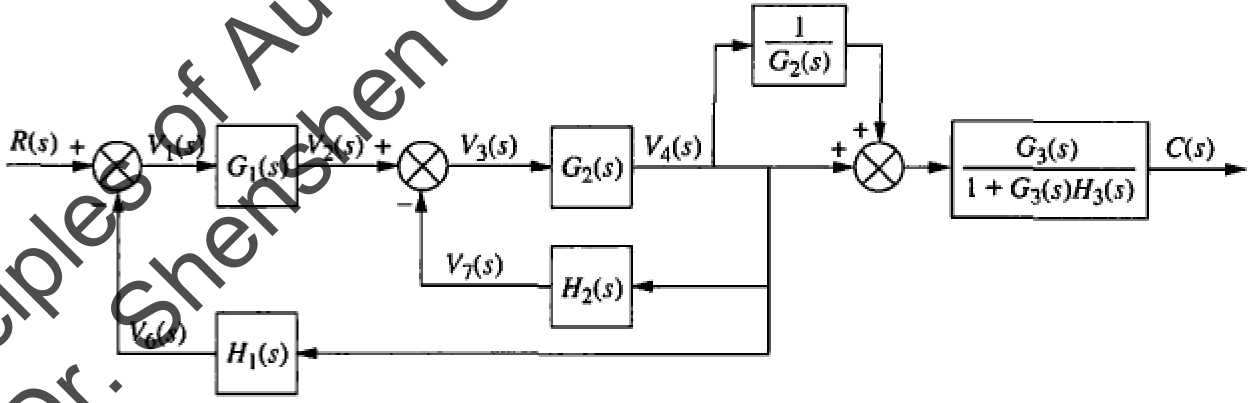


FIGURE 5.11 Block diagram for Example 5.2

SOLUTION: In this example we make use of the equivalent forms shown in Figures 5.7 and 5.8. First, move $G_2(s)$ to the left past the pickoff point to create parallel subsystems, and reduce the feedback system consisting of $G_3(s)$ and $H_3(s)$. This result is shown in Figure 5.12(a).

Second, reduce the parallel pair consisting of $1/G_2(s)$ and unity, and push $G_1(s)$ to the right past the summing junction, creating parallel subsystems in the feedback. These results are shown in Figure 5.12(b).



(*a*)

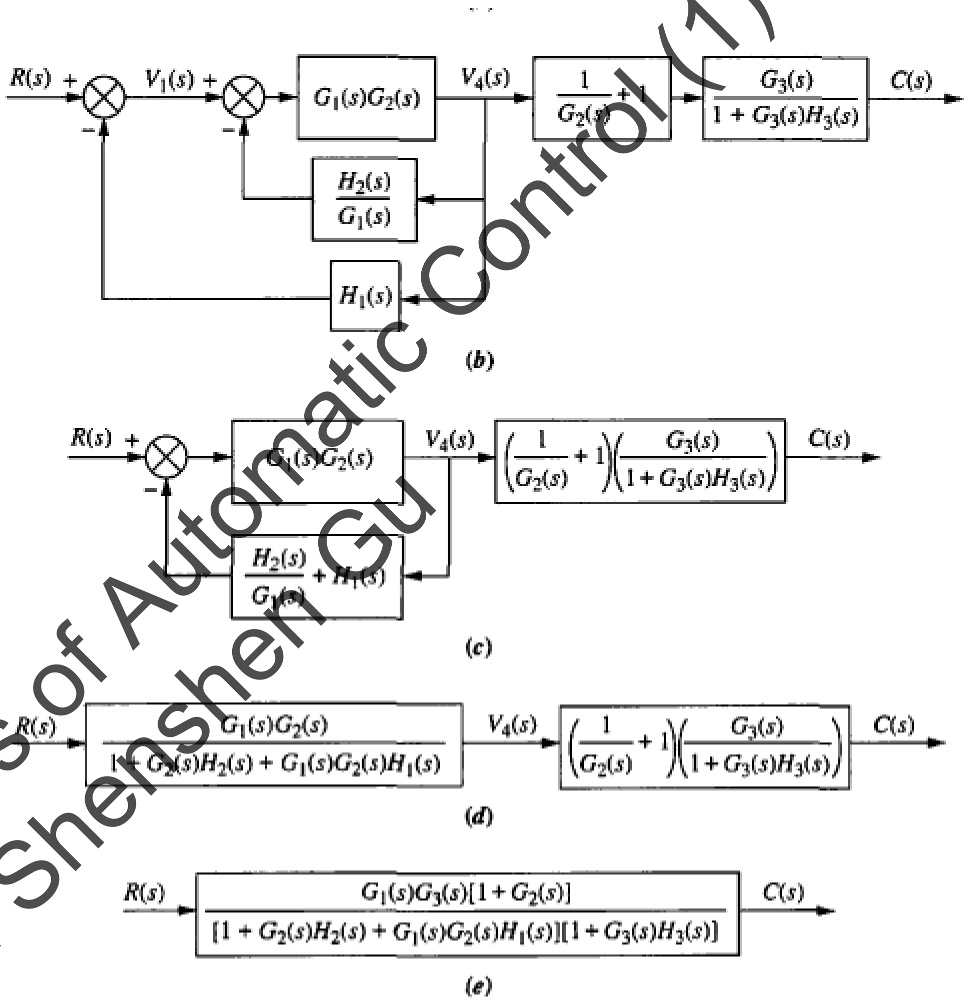
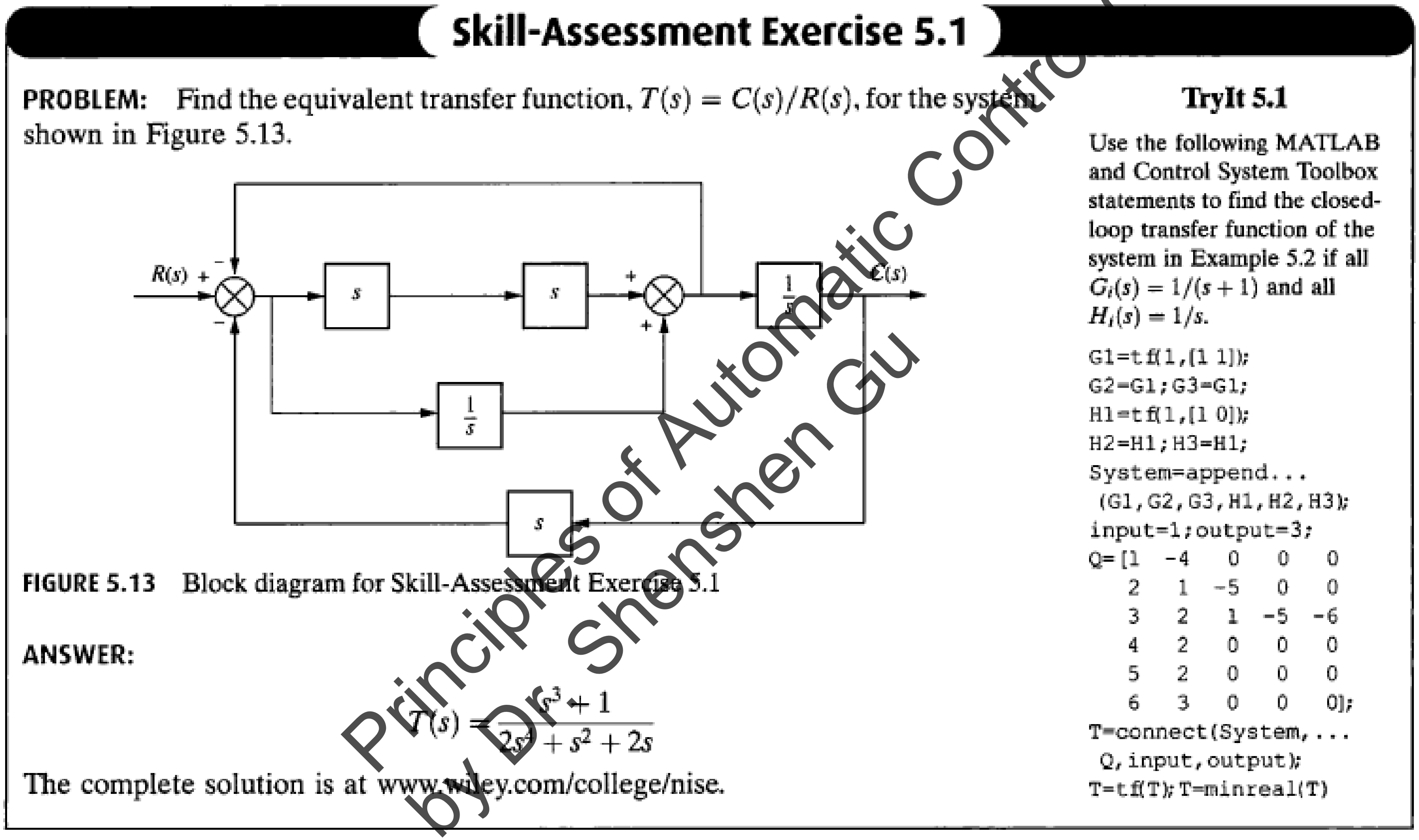


FIGURE 5.12 Steps in the block diagram reduction for Example 5.2



Skill-Assessment Exercise 5.1

PROBLEM: Find the equivalent transfer function, $T(s) = C(s)/R(s)$, for the system shown in Figure 5.13.

FIGURE 5.13 Block diagram for Skill-Assessment Exercise 5.1

ANSWER:

$$T(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$

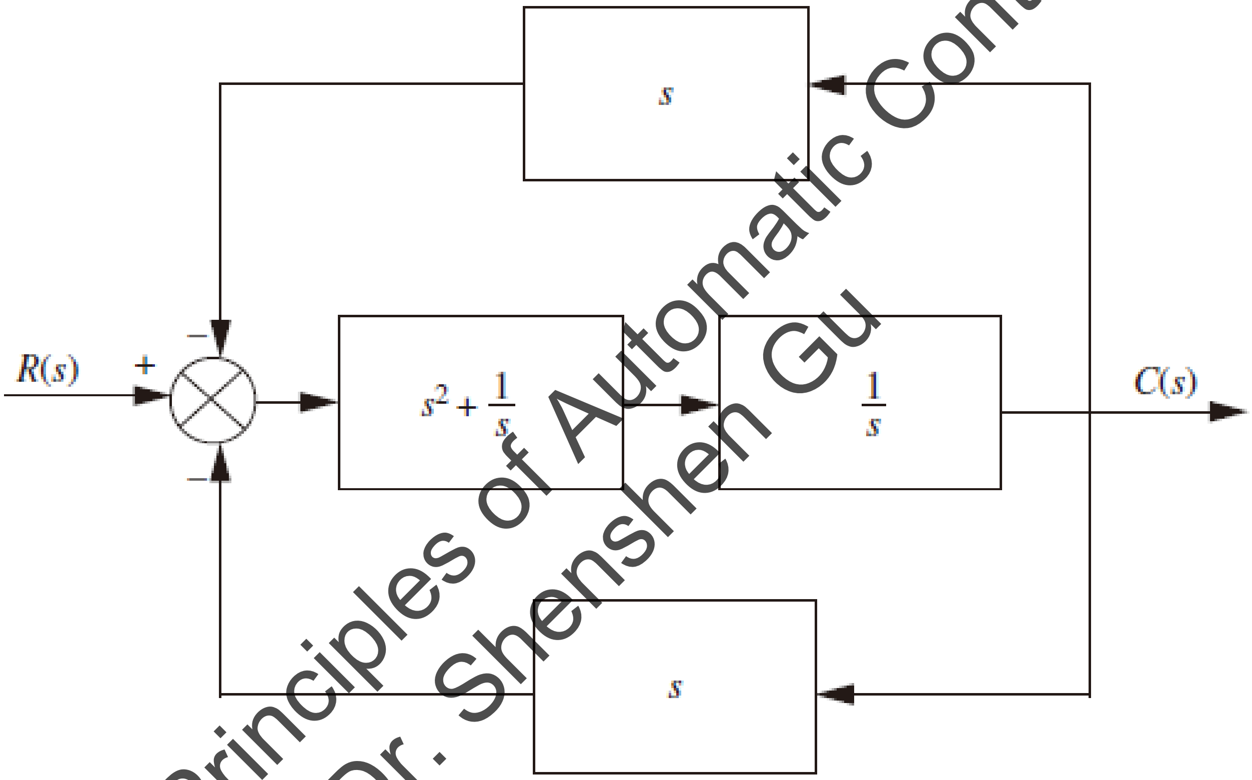
The complete solution is at www.wiley.com/college/nise.

TryIt 5.1

Use the following MATLAB and Control System Toolbox statements to find the closed-loop transfer function of the system in Example 5.2 if all $G_i(s) = 1/(s+1)$ and all $H_i(s) = 1/s$.

```
G1=tf(1,[1 1]);
G2=G1;G3=G1;
H1=tf(1,[1 0]);
H2=H1;H3=H1;
System=append...
 (G1,G2,G3,H1,H2,H3);
input=1;output=3;
Q=[1  -4   0   0   0
   2   1  -5   0   0
   3   2   1  -5  -6
   4   2   0   0   0
   5   2   0   0   0
   6   3   0   0   0];
T=connect(System,...
 Q,input,output);
T=tf(T);T=minreal(T)
```

Combine the parallel blocks in the forward path. Then, push $\frac{1}{s}$ to the left past the pickoff point.



Combine the parallel feedback paths and get $2s$. Then, apply the feedback formula, simplify, and get, $T(s) = \dfrac{s^3 + 1}{2s^4 + s^2 + 2s}$.

Analysis and Design of Feedback Systems

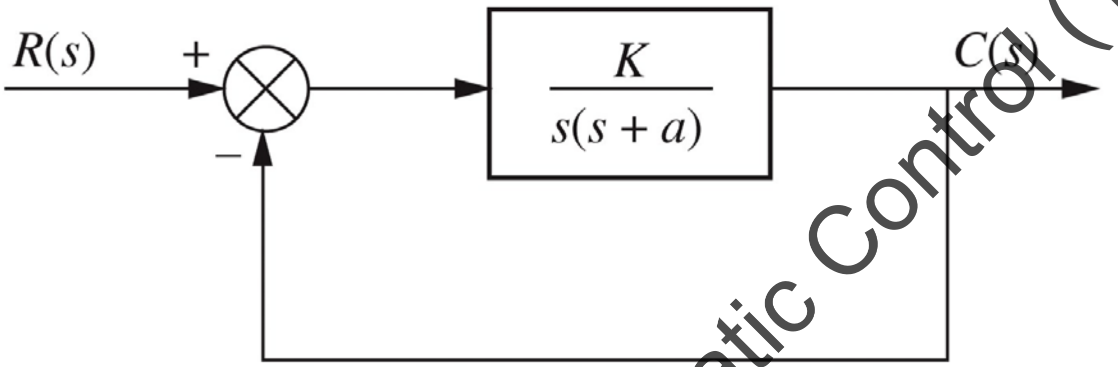
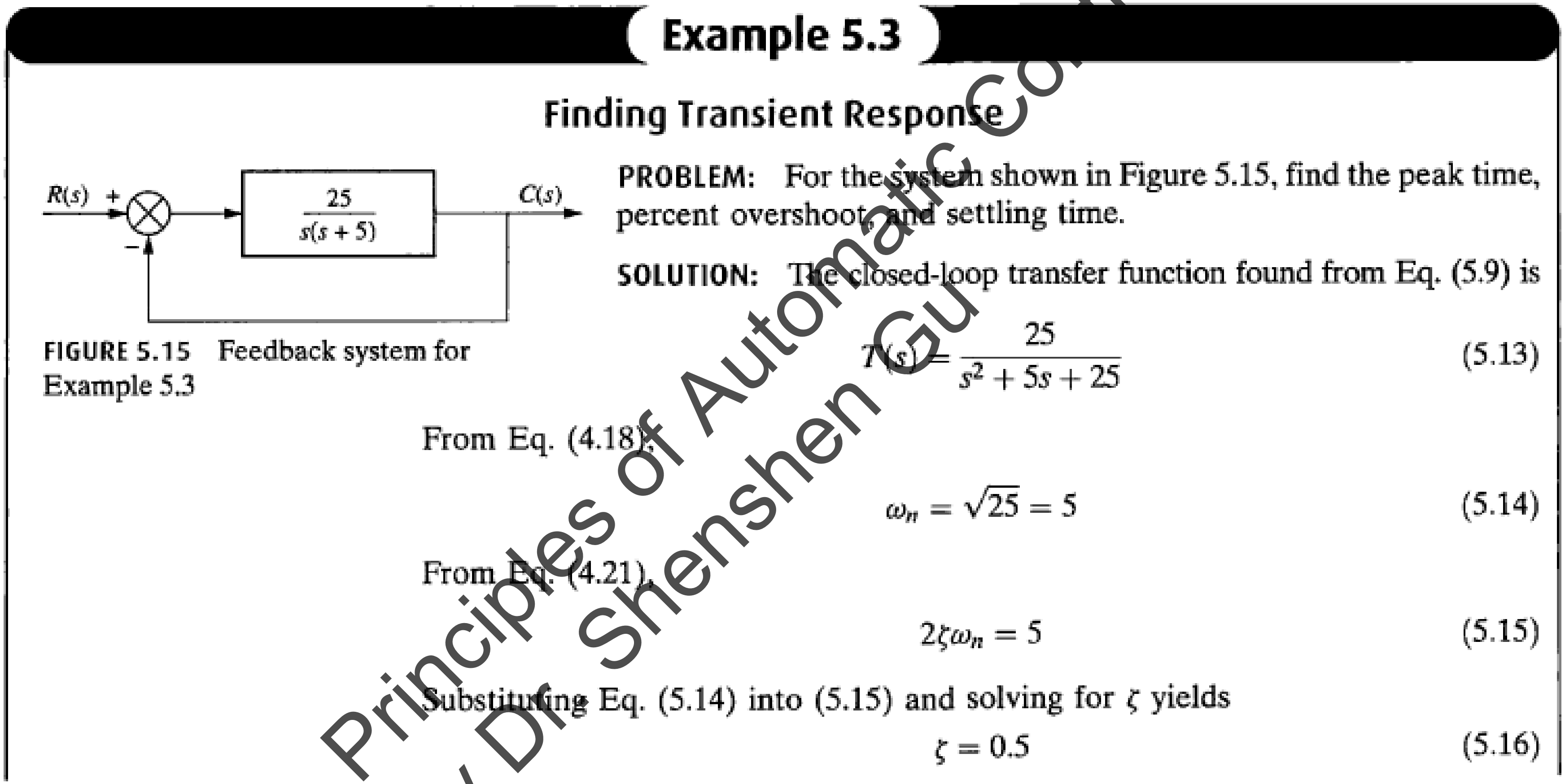


Figure 5.14

$$T(s)=\frac{\dfrac{K}{s(s+a)}}{1+\dfrac{K}{s(s+a)}}=\frac{K}{s^2+as+K}$$

$$\begin{cases} s_{1,2}=-\dfrac{a}{2}\pm\dfrac{\sqrt{a^2-4K}}{2} & ,0<K<\dfrac{a^2}{4} \\ s_{1,2}=-\dfrac{a}{2} & ,K=\dfrac{a^2}{4} \\ s_{1,2}=-\dfrac{a}{2}\pm j\dfrac{\sqrt{4K-a^2}}{2} & ,K>\dfrac{a^2}{4} \end{cases}$$



Example 5.3

Finding Transient Response

FIGURE 5.15 Feedback system for Example 5.3

PROBLEM: For the system shown in Figure 5.15, find the peak time, percent overshoot, and settling time.

SOLUTION: The closed-loop transfer function found from Eq. (5.9) is

$$T(s) = \frac{25}{s^2 + 5s + 25} \quad (5.13)$$

From Eq. (4.18),

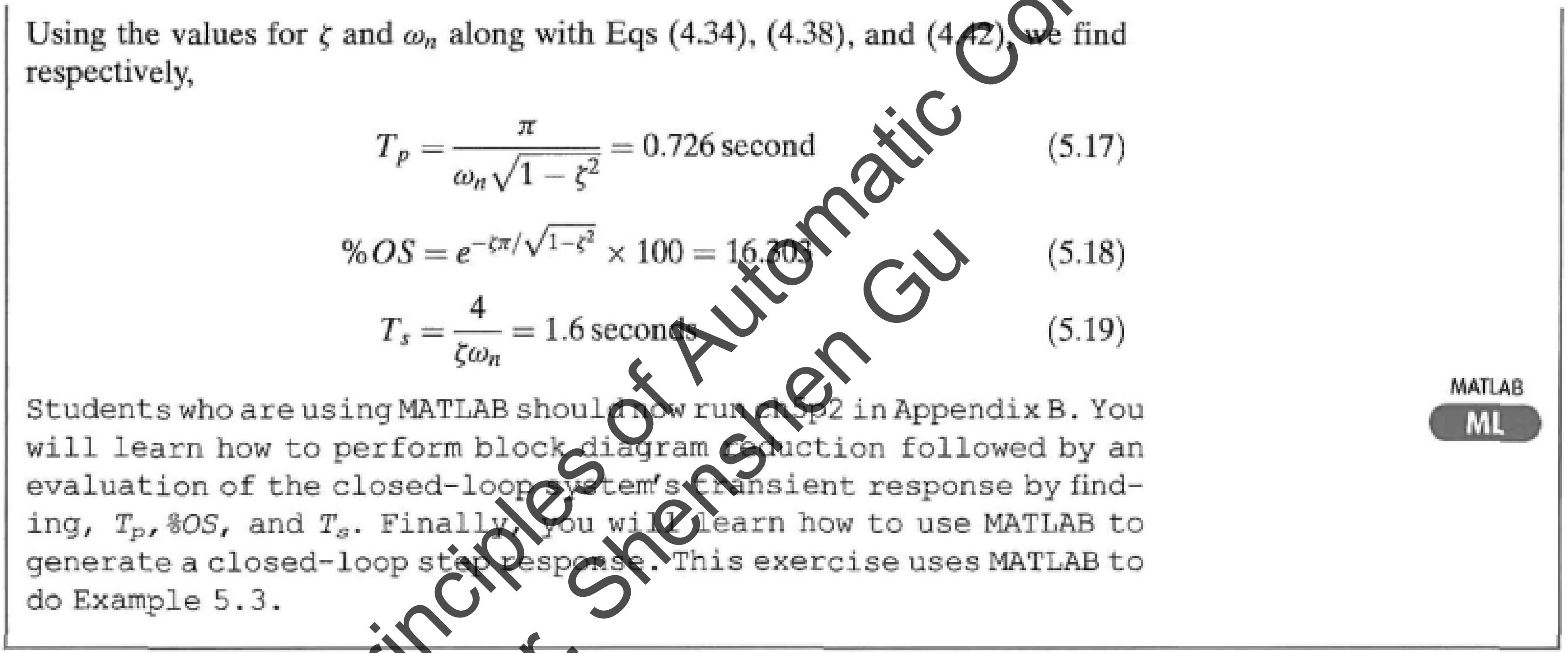
$$\omega_n = \sqrt{25} = 5 \quad (5.14)$$

From Eq. (4.21),

$$2\zeta\omega_n = 5 \quad (5.15)$$

Substituting Eq. (5.14) into (5.15) and solving for ζ yields

$$\zeta = 0.5 \quad (5.16)$$



Using the values for ζ and ω_n along with Eqs (4.34), (4.38), and (4.42), we find respectively,

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.726 \text{ second} \tag{5.17}$$

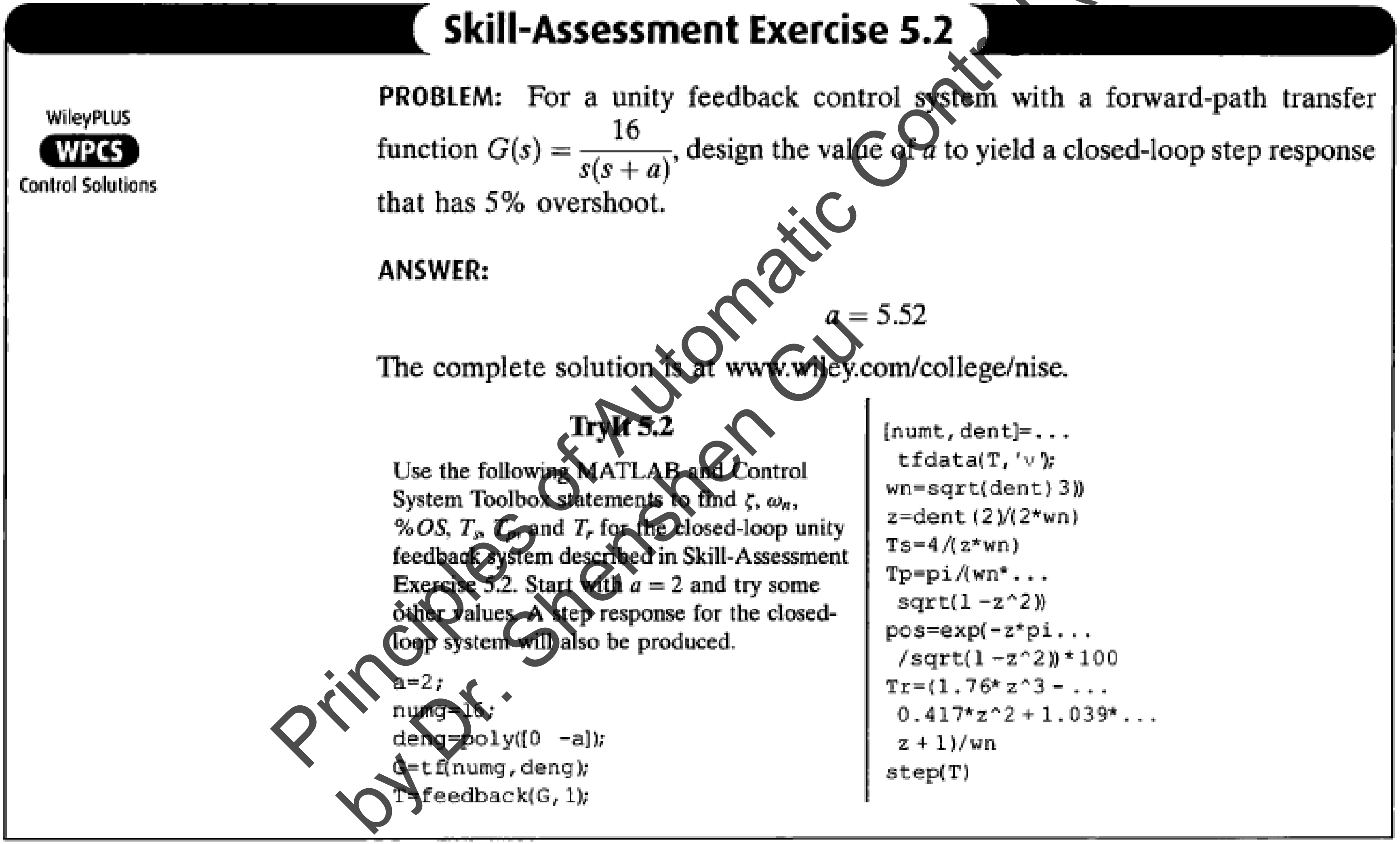
$$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 16.303 \tag{5.18}$$

$$T_s = \frac{4}{\zeta\omega_n} = 1.6 \text{ seconds} \tag{5.19}$$

MATLAB

ML

Students who are using MATLAB should now run ch5p2 in Appendix B. You will learn how to perform block diagram reduction followed by an evaluation of the closed-loop system's transient response by finding, T_p, $\%OS$, and T_s. Finally, you will learn how to use MATLAB to generate a closed-loop step response. This exercise uses MATLAB to do Example 5.3.



Skill-Assessment Exercise 5.2

WileyPLUS
WPCS
Control Solutions

PROBLEM: For a unity feedback control system with a forward-path transfer function $G(s) = \frac{16}{s(s+a)}$, design the value of a to yield a closed-loop step response that has 5% overshoot.

ANSWER:

$$a = 5.52$$

The complete solution is at www.wiley.com/college/nise.

TryIt 5.2

Use the following MATLAB and Control System Toolbox statements to find ζ, ω_n, $\%OS$, T_s, T_p, and T_r for the closed-loop unity feedback system described in Skill-Assessment Exercise 5.2. Start with $a = 2$ and try some other values. A step response for the closed-loop system will also be produced.

```
a=2;
numg=16;
deng=poly([0 -a]);
G=tf(numg,deng);
T=feedback(G,1);
[numt,dent]=...
 tfdata(T,'v');
wn=sqrt(dent(3))
z=dent(2)/(2*wn)
Ts=4/(z*wn)
Tp=pi/(wn*...
 sqrt(1-z^2))
pos=exp(-z*pi...
 /sqrt(1-z^2))*100
Tr=(1.76*z^3-...
 0.417*z^2+1.039*...
 z+1)/wn
step(T)
```

Find the closed-loop transfer function, $T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{16}{s^2+as+16}$, where and $G(s) = \frac{16}{s(s+a)}$ and $H(s) = 1$. Thus, $\omega_n = 4$ and $2\zeta\omega_n = a$, from which $\zeta = \frac{a}{8}$.

But, for 5% overshoot, $\zeta = \frac{-\ln\left(\frac{\%}{100}\right)}{\sqrt{\pi^2+\ln^2\left(\frac{\%}{100}\right)}} = 0.69$. Since, $\zeta = \frac{a}{8}$, $a = 5.52$.

Signal-Flow Graphs

- Signal-flow graphs are an alternative to block diagrams;
- A signal-flow graph consists only of ***branches***, which represent systems, and ***nodes***, which represent signals;

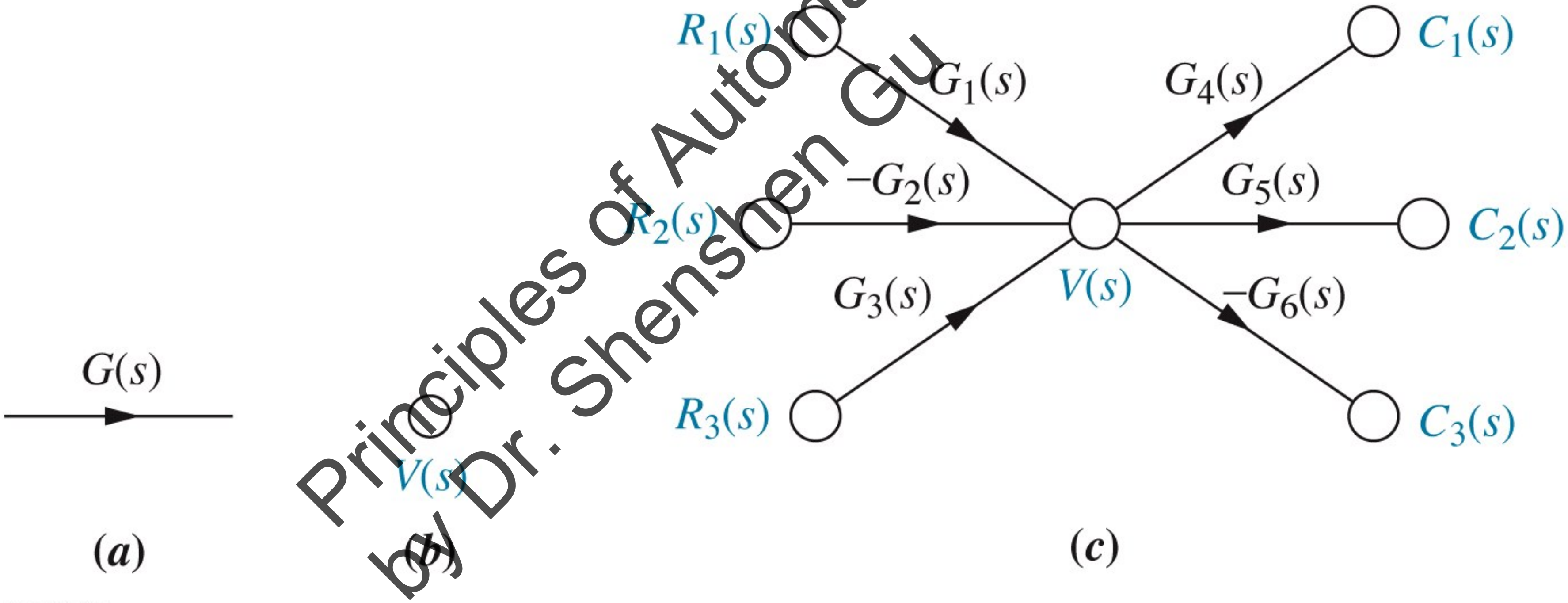
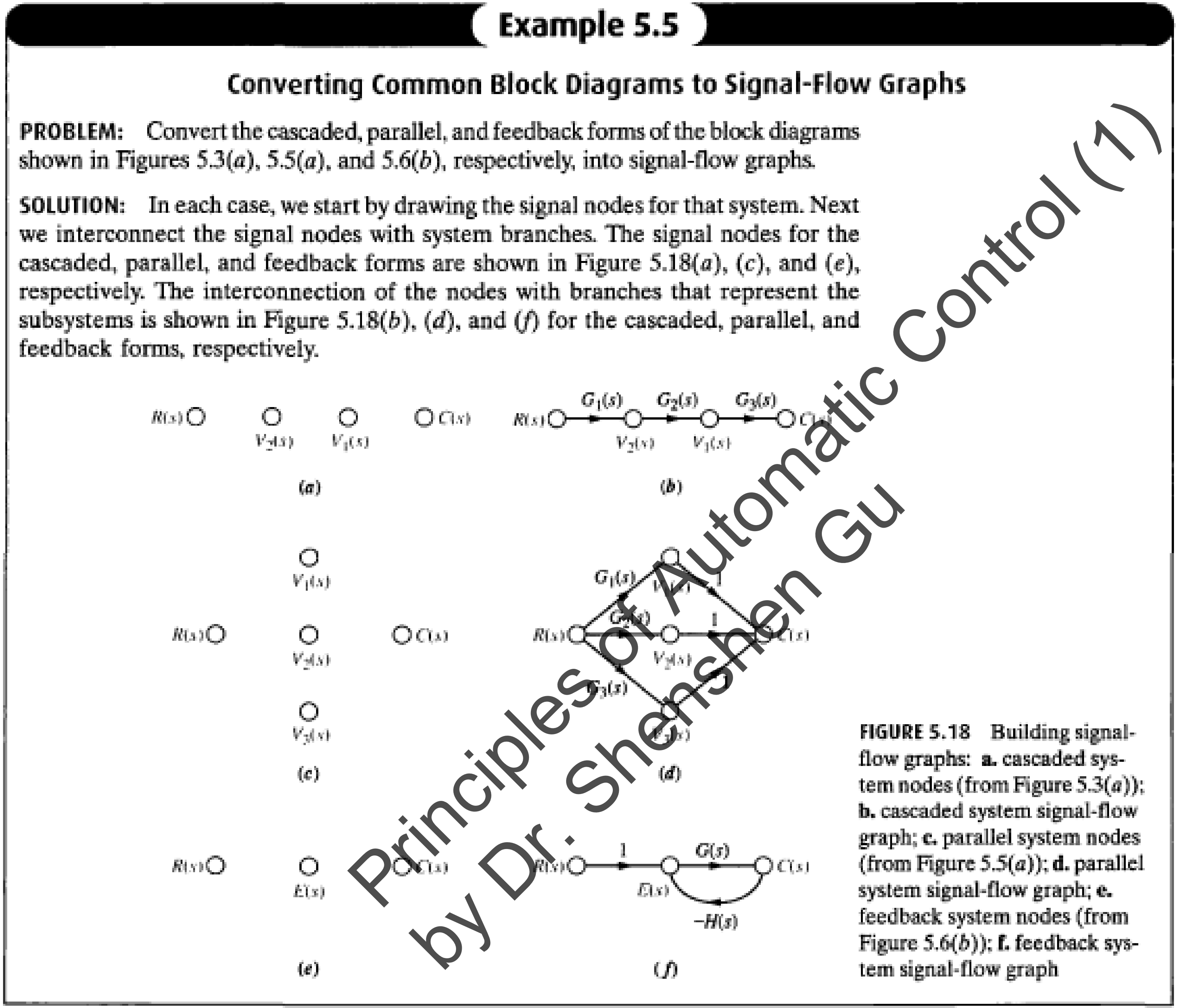


Figure 5.17



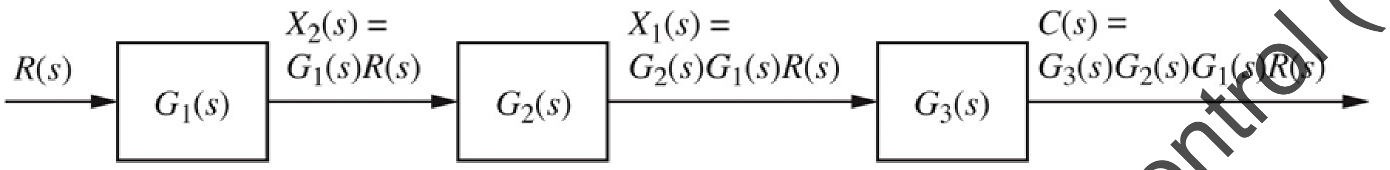
Example 5.5

Converting Common Block Diagrams to Signal-Flow Graphs

PROBLEM: Convert the cascaded, parallel, and feedback forms of the block diagrams shown in Figures 5.3(*a*), 5.5(*a*), and 5.6(*b*), respectively, into signal-flow graphs.

SOLUTION: In each case, we start by drawing the signal nodes for that system. Next we interconnect the signal nodes with system branches. The signal nodes for the cascaded, parallel, and feedback forms are shown in Figure 5.18(*a*), (*c*), and (*e*), respectively. The interconnection of the nodes with branches that represent the subsystems is shown in Figure 5.18(*b*), (*d*), and (*f*) for the cascaded, parallel, and feedback forms, respectively.

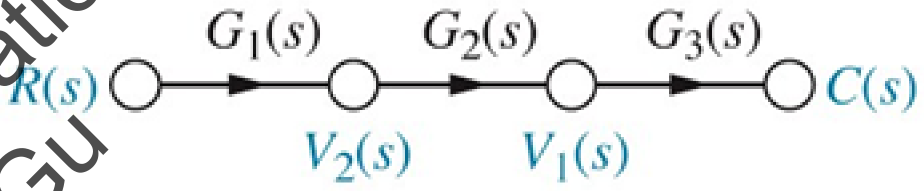
FIGURE 5.18 Building signal-flow graphs: **a.** cascaded system nodes (from Figure 5.3(*a*)); **b.** cascaded system signal-flow graph; **c.** parallel system nodes (from Figure 5.5(*a*)); **d.** parallel system signal-flow graph; **e.** feedback system nodes (from Figure 5.6(*b*)); **f.** feedback system signal-flow graph


R(s)
G1(s)
X2(s) =
G1(s)R(s)
G2(s)
X1(s) =
G2(s)G1(s)R(s)
G3(s)
C(s) =
G3(s)G2(s)G1(s)R(s)

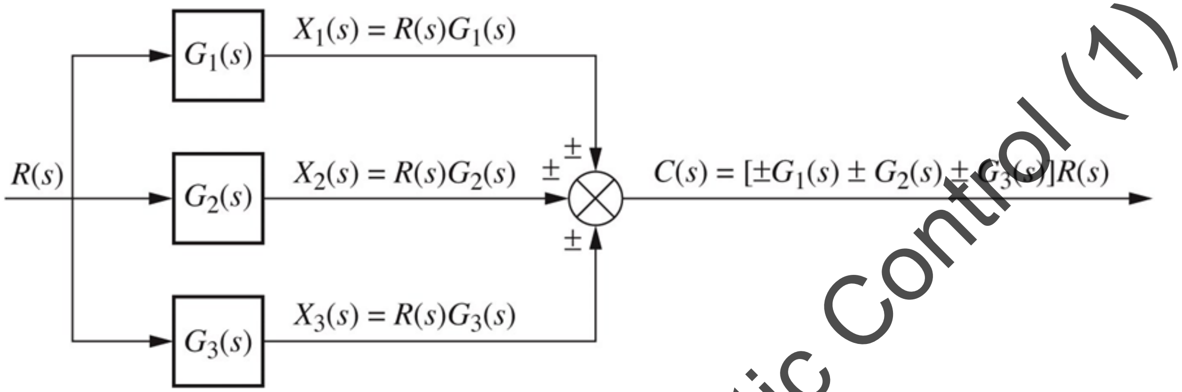
(*a*)

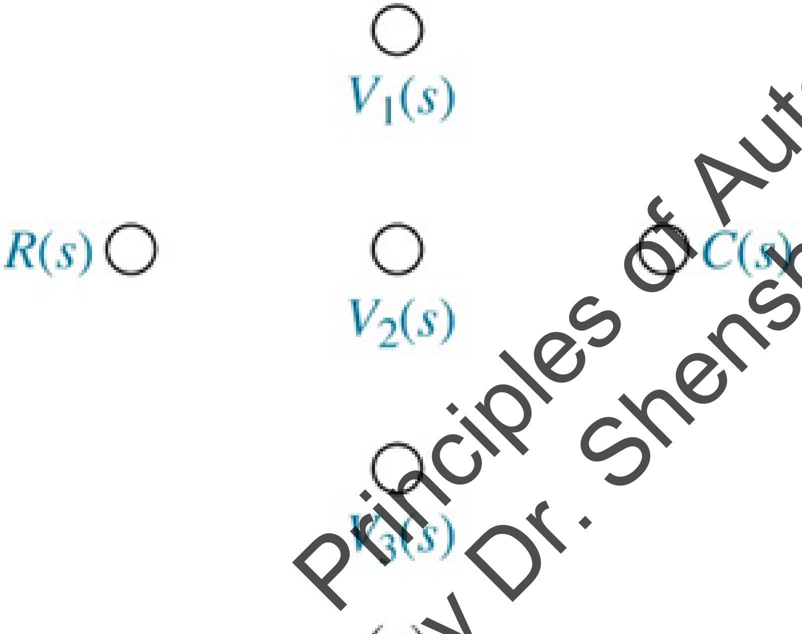

R(s)
V2(s)
V1(s)
C(s)

(*a*)

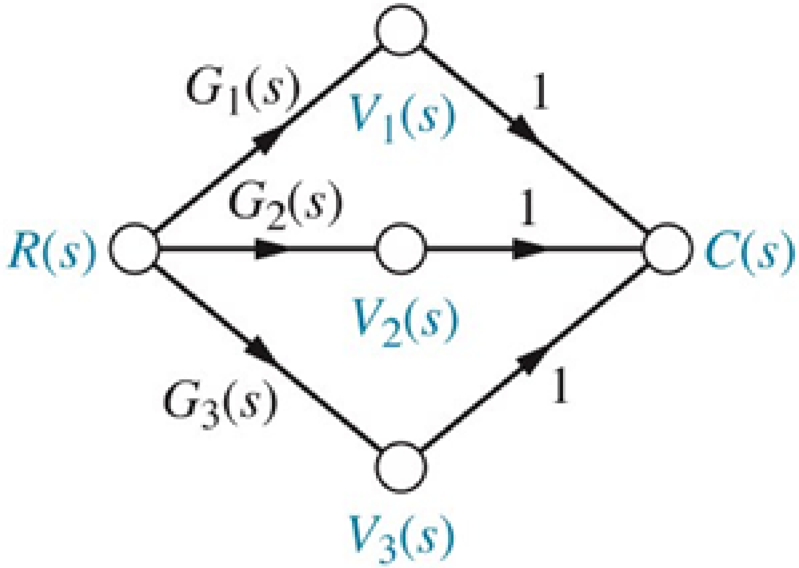

G1(s)
G2(s)
G3(s)
R(s)
V2(s)
V1(s)
C(s)

(*b*)

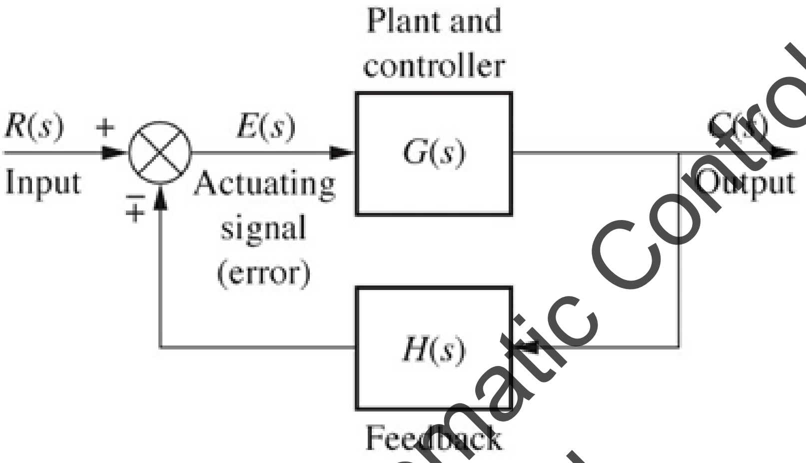

G_1(s)
G_2(s)
G_3(s)
R(s)
X_1(s) = R(s)G_1(s)
X_2(s) = R(s)G_2(s)
X_3(s) = R(s)G_3(s)
±
±
±
C(s) = [±G_1(s) ± G_2(s) ± G_3(s)]R(s)

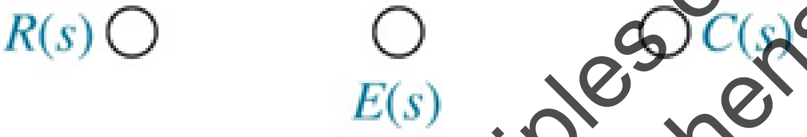

R(s)
V_1(s)
V_2(s)
V_3(s)
C(s)

(c)

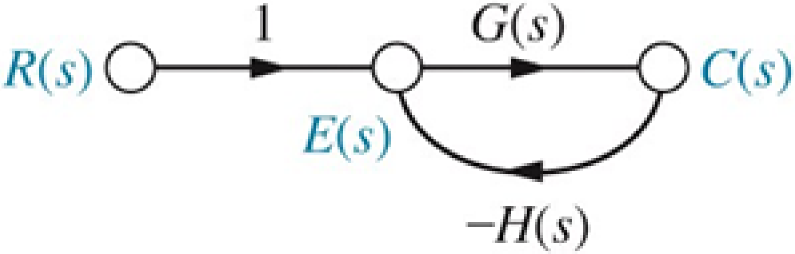

R(s)
G_1(s)
G_2(s)
G_3(s)
V_1(s)
V_2(s)
V_3(s)
1
1
1
C(s)

(d)

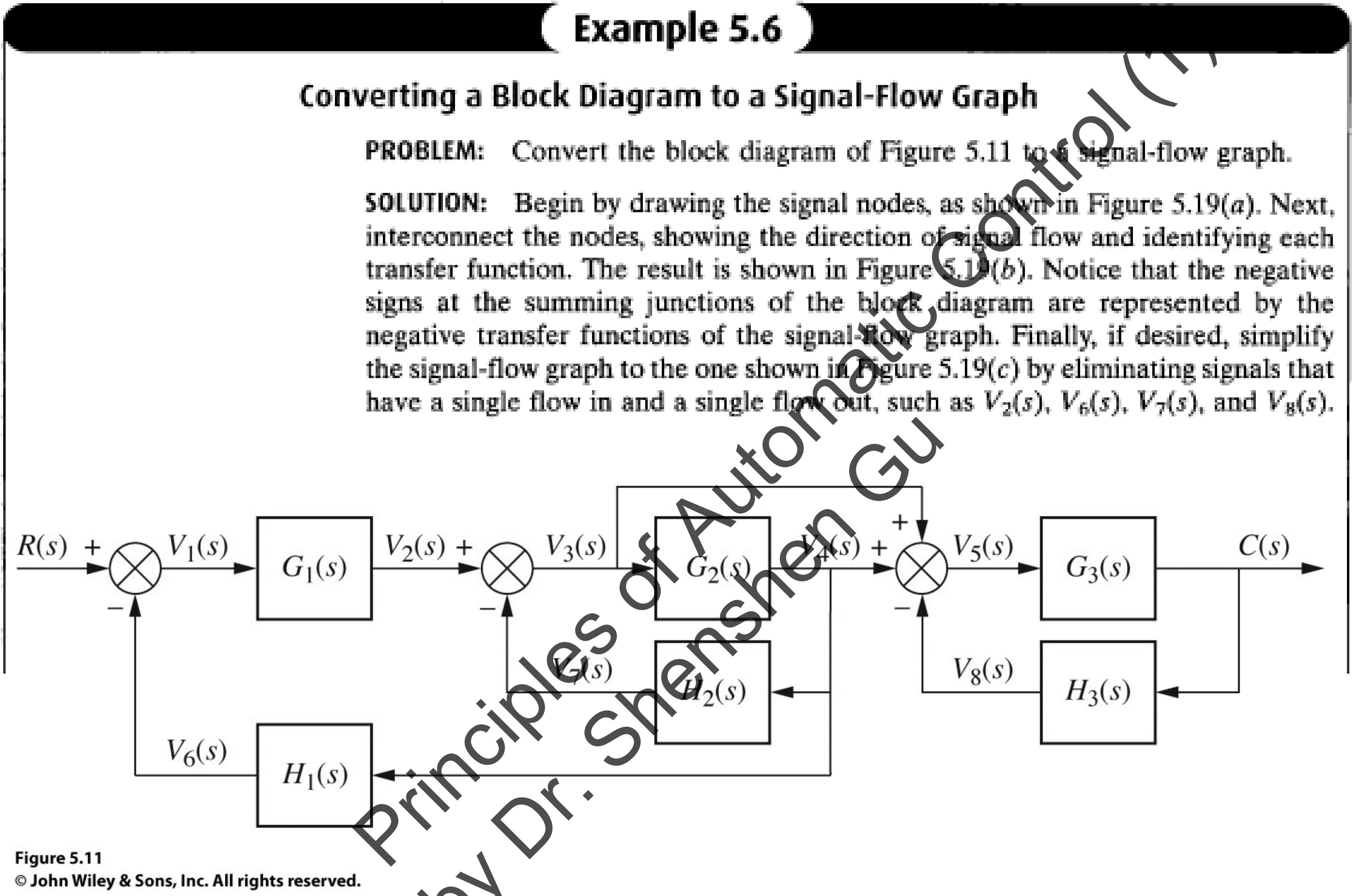

Plant and controller
R(s)
+
Input
∓
E(s)
Actuating signal (error)
G(s)
C(s)
Output
H(s)
Feedback


R(s)
E(s)
C(s)

(e)


1
G(s)
R(s)
E(s)
C(s)
−H(s)

(f)



Example 5.6

Converting a Block Diagram to a Signal-Flow Graph

PROBLEM: Convert the block diagram of Figure 5.11 to a signal-flow graph.

SOLUTION: Begin by drawing the signal nodes, as shown in Figure 5.19(a). Next, interconnect the nodes, showing the direction of signal flow and identifying each transfer function. The result is shown in Figure 5.19(b). Notice that the negative signs at the summing junctions of the block diagram are represented by the negative transfer functions of the signal-flow graph. Finally, if desired, simplify the signal-flow graph to the one shown in Figure 5.19(c) by eliminating signals that have a single flow in and a single flow out, such as $V_2(s)$, $V_6(s)$, $V_7(s)$, and $V_8(s)$.

Figure 5.11

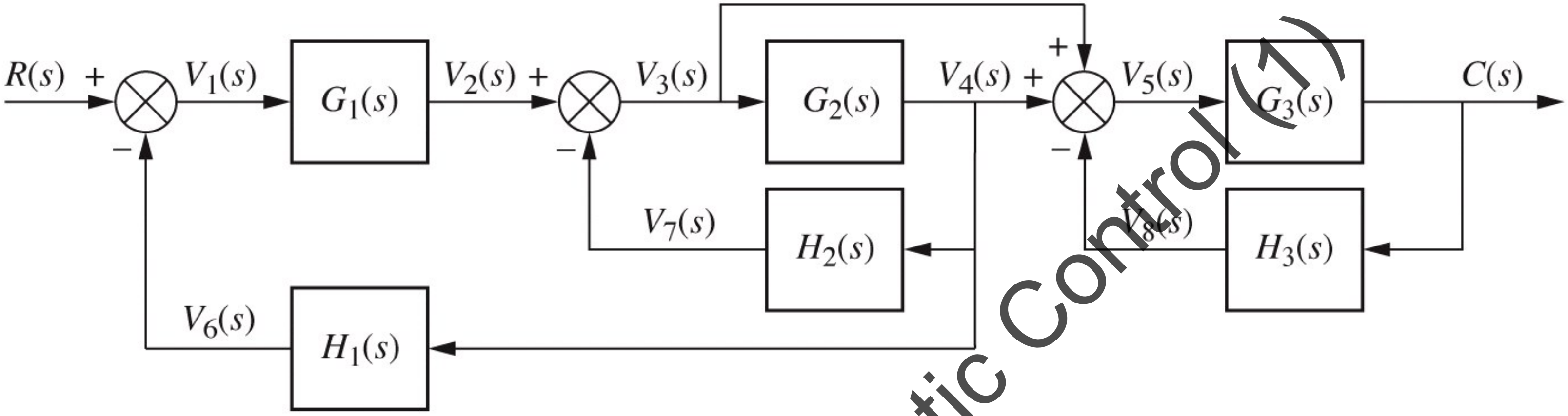

R(s)
+
−
V1(s)
G1(s)
V2(s)
+
−
V3(s)
G2(s)
V4(s)
+
+
−
V5(s)
G3(s)
C(s)
V7(s)
H2(s)
V8(s)
H3(s)
V6(s)
H1(s)

Figure 5.11


R(s)
V1(s)
V2(s)
V3(s)
V4(s)
V5(s)
C(s)
V6(s)
V7(s)
V8(s)

(a)

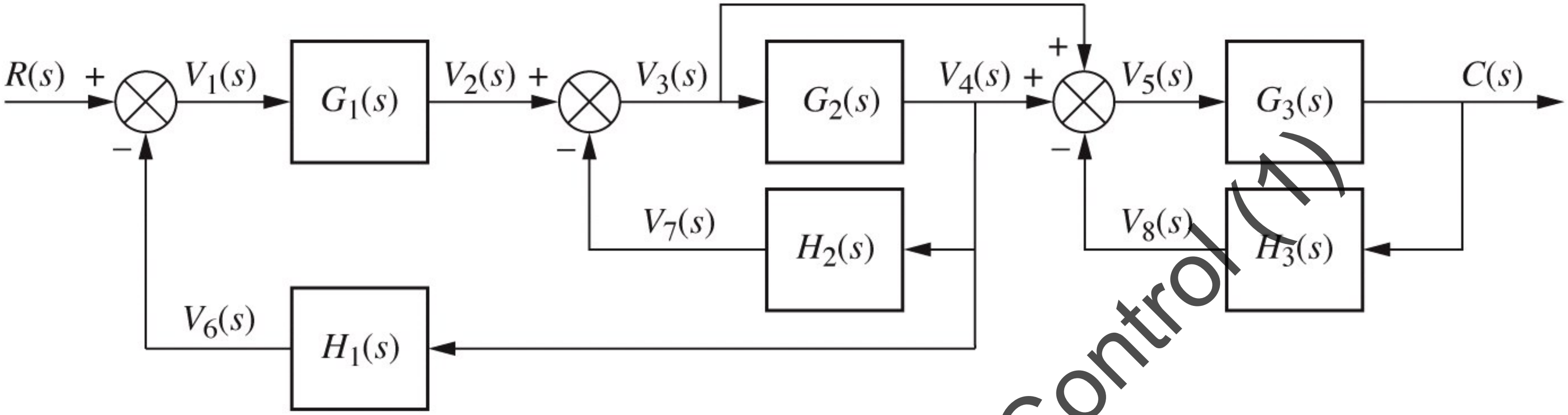


Figure 5.11
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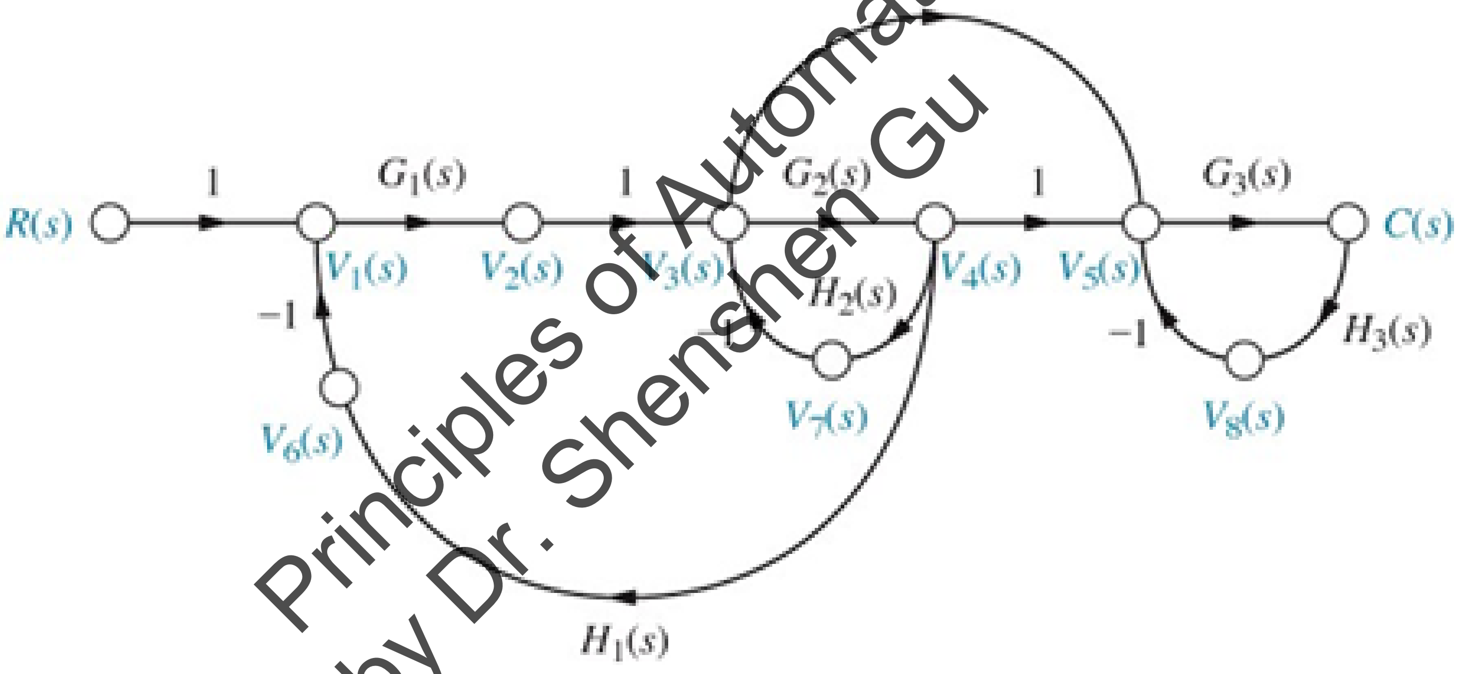


(b)

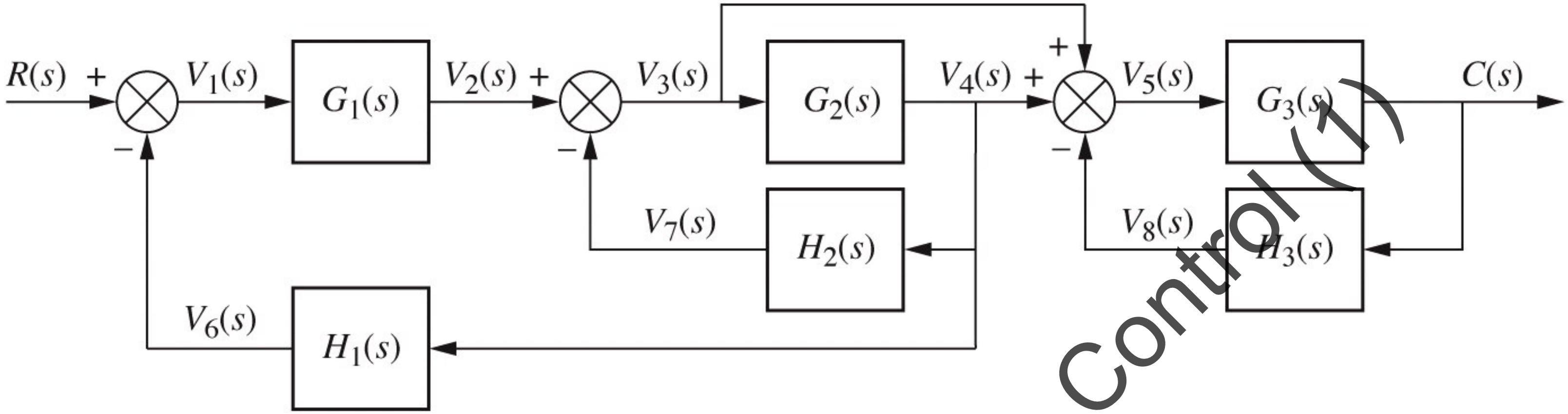
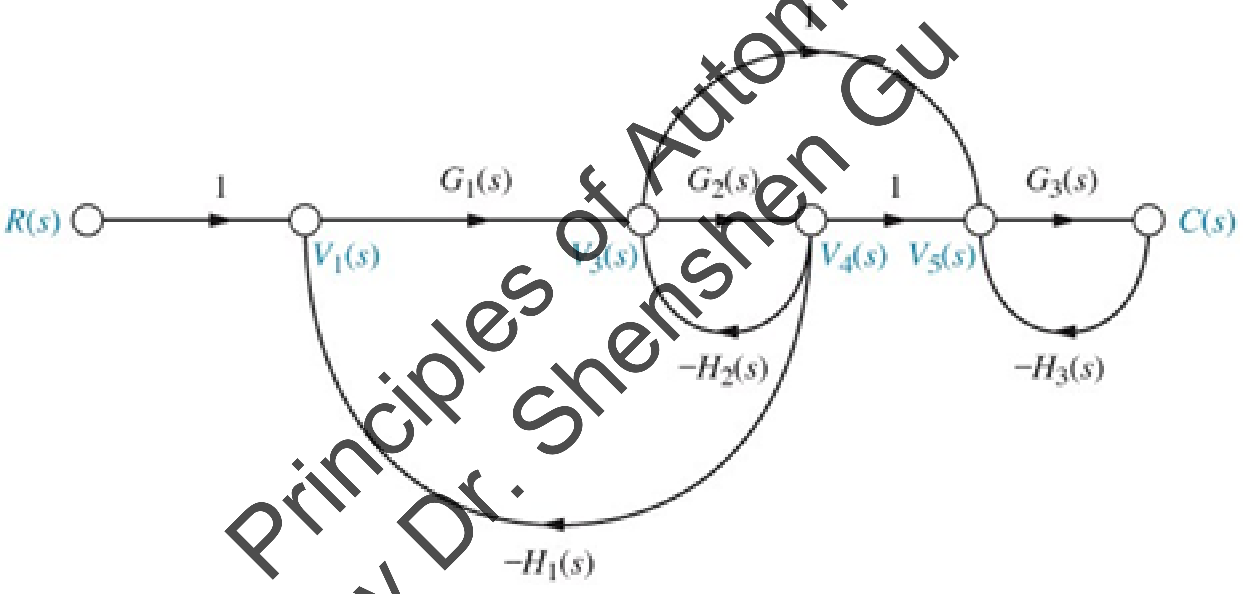

R(s)
+
V_1(s)
G_1(s)
V_2(s)
+
V_3(s)
G_2(s)
V_4(s)
+
+
V_5(s)
G_3(s)
C(s)
−
−
−
V_7(s)
H_2(s)
V_8(s)
H_3(s)
V_6(s)
H_1(s)

Figure 5.11


1
1
G_1(s)
G_2(s)
1
G_3(s)
R(s)
C(s)
V_1(s)
V_3(s)
V_4(s)
V_5(s)
−H_2(s)
−H_3(s)
−H_1(s)

(c)

Mason’s Rule

- Block diagram reduction technique requires successive application of fundamental relationships in order to arrive at the system transfer function.
- Mason’s rule for reducing a signal-flow graph to a single transfer function requires the application of one formula.
- The formula was derived by S.J. Mason in “Feedback Theory-Some properties of Signal-Flow Graphs” in 1953.

Definitions

- ***Loop gain***. The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once.

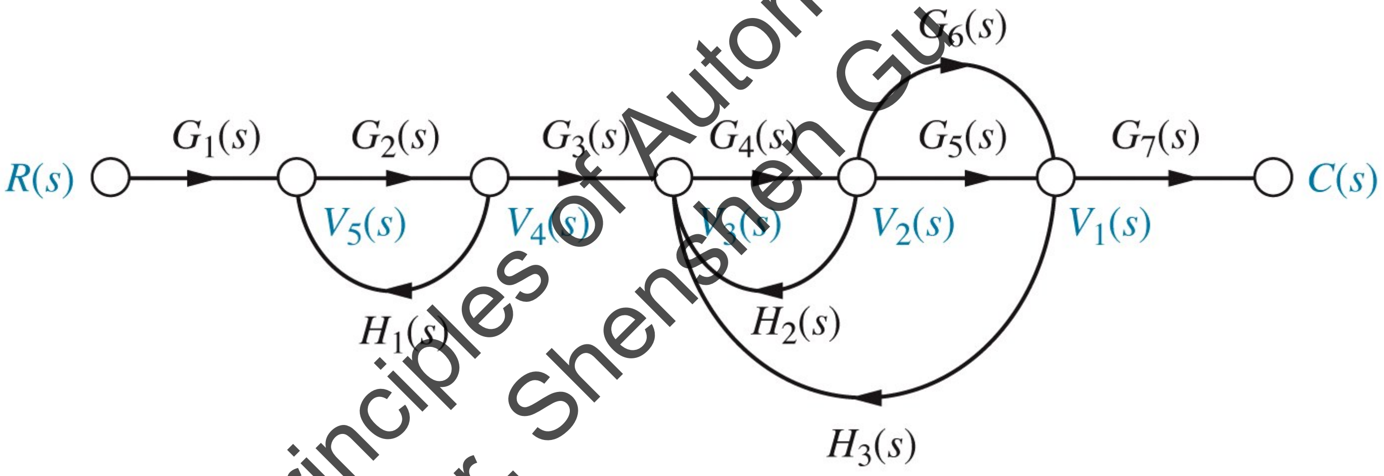


Figure 5.20
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- Four loop gains:
  - 1. $G_2(s)H_1(s)$
  - 2. $G_4(s)H_2(s)$
  - 3. $G_4(s)G_5(s)H_3(s)$
  - 4. $G_4(s)G_6(s)H_3(s)$

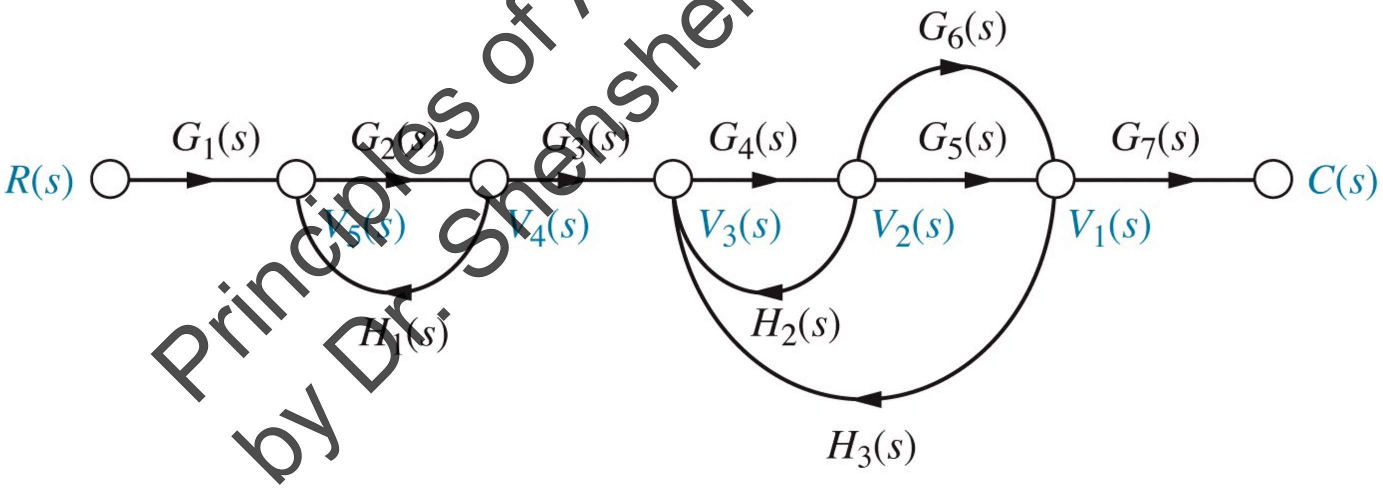

Figure 5.20

- ***Forward-path gain***. The product of gains found by traversing a path form the input node to the output node of the signal-flow graph in the direction of signal flow.

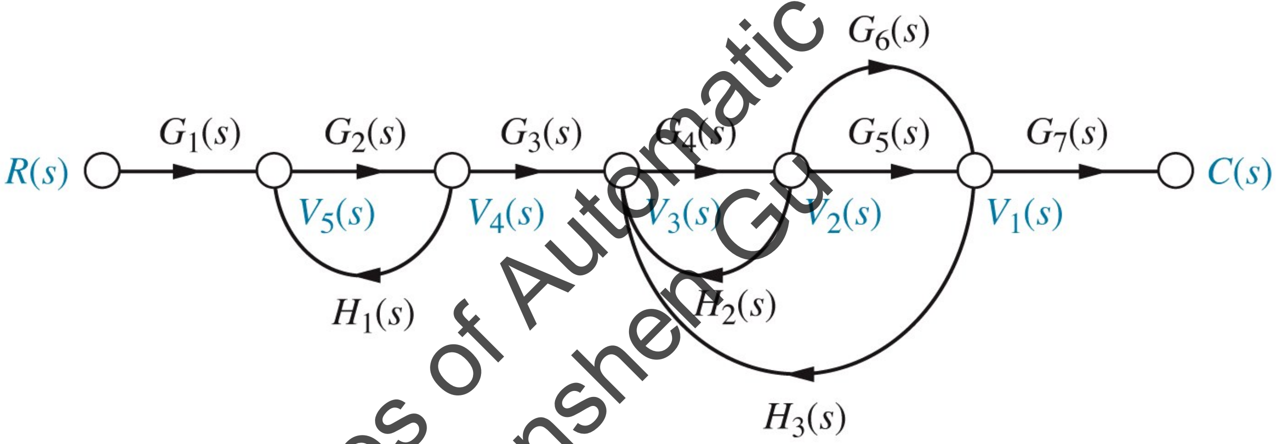

Figure 5.20
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- Two forward-path gains:
  - 1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$
  - 2. $\mathrm{G}_1(\mathrm{s})\mathrm{G}_2(\mathrm{s})\mathrm{G}_3(\mathrm{s})\mathrm{G}_4(\mathrm{s})\mathrm{G}_6(\mathrm{s})\mathrm{G}_7(\mathrm{s})$

- ***Nontouching loops***. Loops that do not have any nodes in common.
- ***Nontouching-loop gain***. The product of loop gains from nontouching loops taken two, three, four, or more at a time.

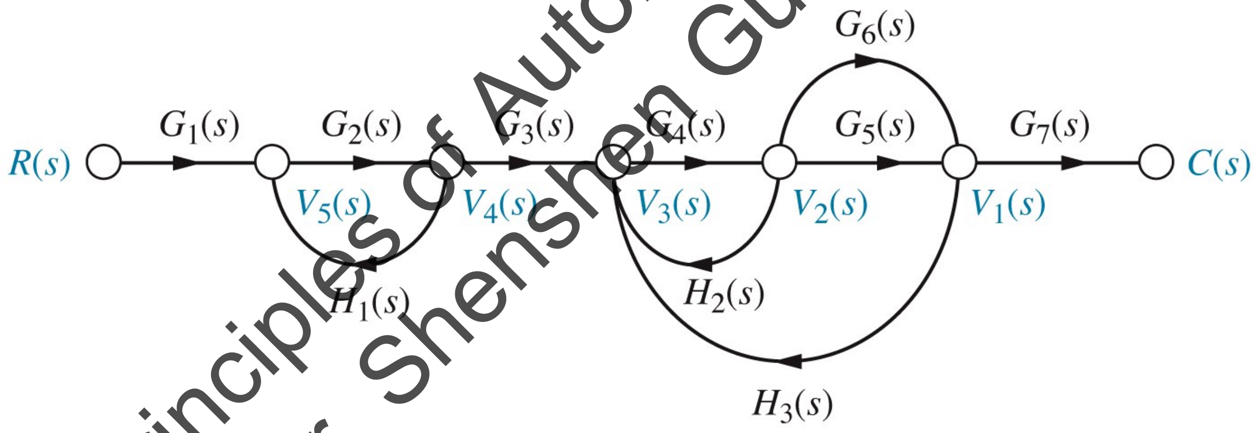

Figure 5.20
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- Loop $G_2(s)H_1(s)$ does not touch loops $G_4(s)H_2(s)$, $G_4(s)G_5(s)H_3(s)$, and $G_4(s)G_6(s)H_3(s)$.
- Three nontouching-loop gains taken two at a time:
  - $[G_2(s)H_1(s)][G_4(s)H_2(s)]$
  - $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$
  - $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

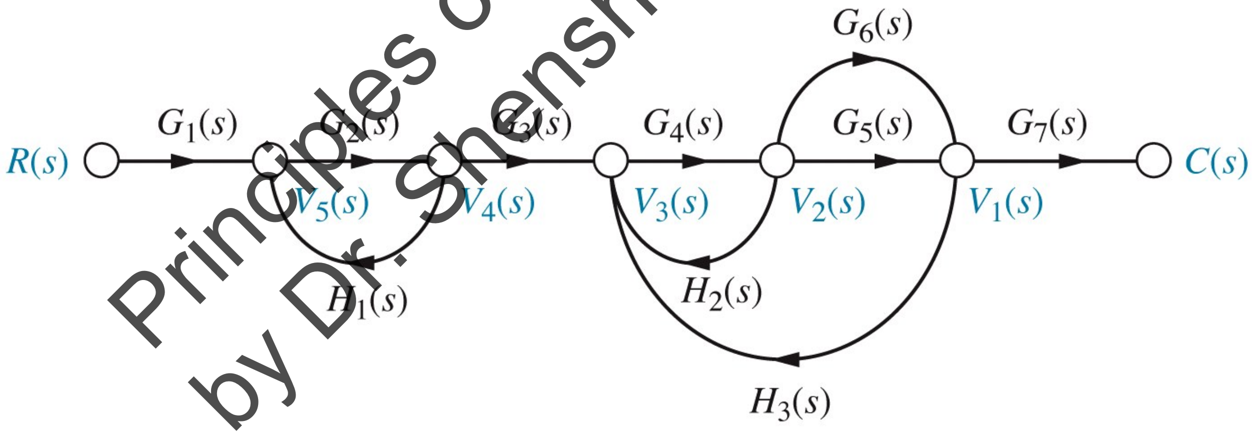


Figure 5.20

Mason's Rule

$$G(s)=\frac{C(s)}{R(s)}=\frac{\sum_k T_k\Delta_k}{\Delta}$$

- *k* = number of forward paths;
- T_k = the *k*th forward-path gain;
- *Δ*=1 - ∑loop gains + ∑nontouching-loop gains taken two at a time - ∑nontouching-loop gains taken three at a time + ∑nontouching-loop gains taken four at a time - …
- Δ_k=*Δ* - ∑loop gain terms in Δ that touch the *k*th forward path. In other words, Δ_k is formed by eliminating from *Δ* those loop gains that touch the *k*th forward path.
- **Notice the alternating signs for the components of *Δ*.**

Example 5.7

Transfer Function via Mason's Rule

PROBLEM: Find the transfer function, $C(s)/R(s)$, for the signal-flow graph in Figure 5.21.

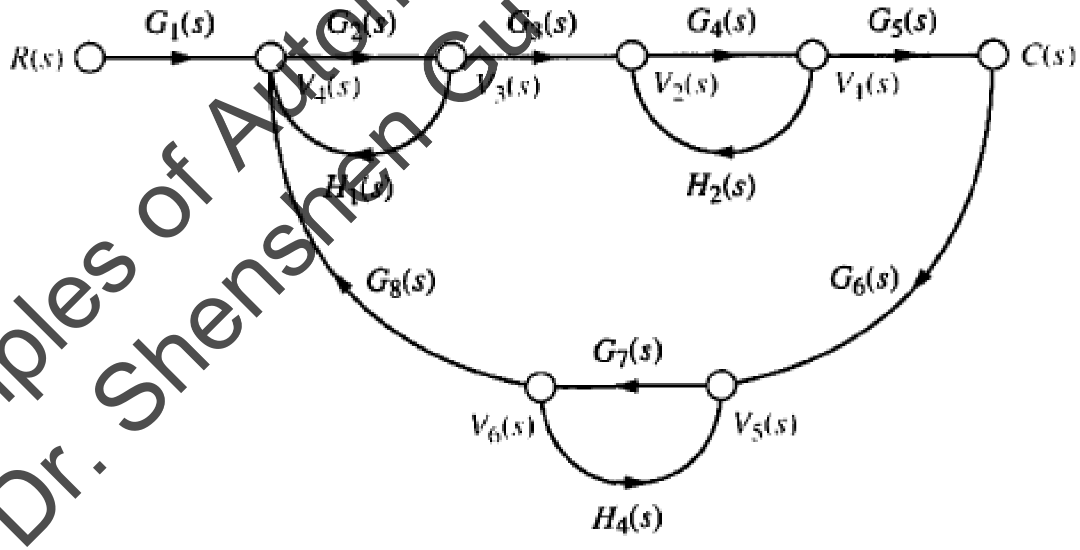
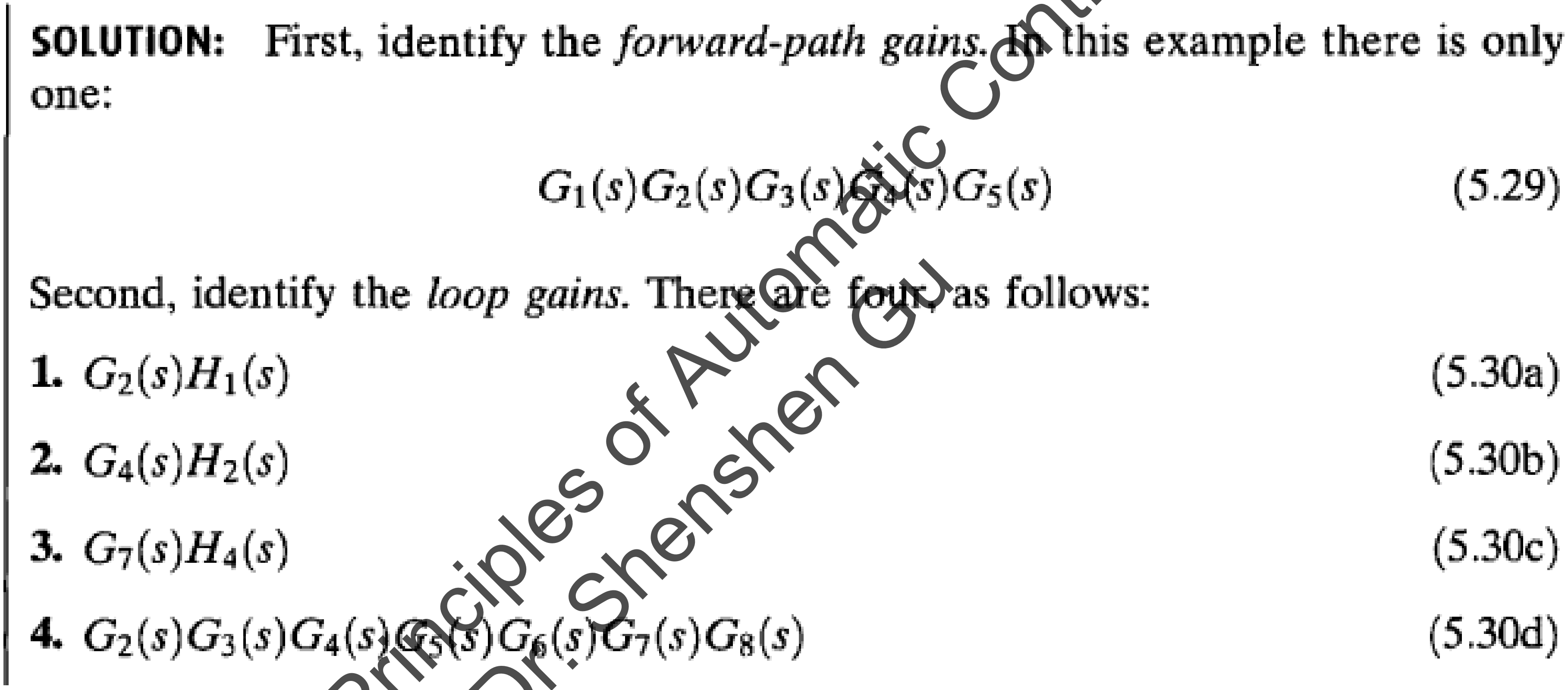


FIGURE 5.21 Signal-flow graph for Example 5.7



SOLUTION: First, identify the *forward-path gains*. In this example there is only one:

$$G_1(s)G_2(s)G_3(s)G_4(s)G_5(s) \tag{5.29}$$

Second, identify the *loop gains*. There are four, as follows:

1. $G_2(s)H_1(s)$ (5.30a)
2. $G_4(s)H_2(s)$ (5.30b)
3. $G_7(s)H_4(s)$ (5.30c)
4. $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$ (5.30d)

Third, identify the *nontouching loops taken two at a time*. From Eqs. (5.30) and Figure 5.21, we can see that loop 1 does not touch loop 2, loop 1 does not touch loop 3, and loop 2 does not touch loop 3. Notice that loops 1, 2, and 3 all touch loop 4. Thus, the combinations of nontouching loops taken two at a time are as follows:

$$\text{Loop 1 and loop 2}: \ G_2(s)H_1(s)G_4(s)H_2(s) \tag{5.31a}$$

$$\text{Loop 1 and loop 3}: \ G_2(s)H_1(s)G_7(s)H_4(s) \tag{5.31b}$$

$$\text{Loop 2 and loop 3}: \ G_4(s)H_2(s)G_7(s)H_4(s) \tag{5.31c}$$

Finally, the *nontouching loops taken three at a time* are as follows:

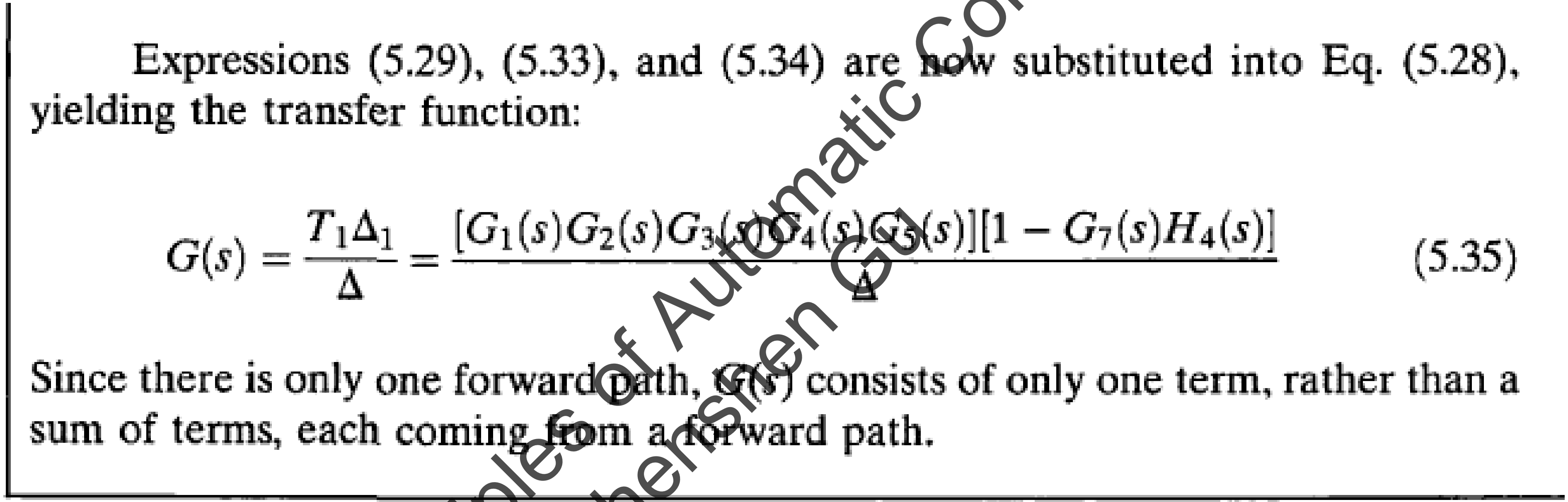
$$\text{Loops 1, 2, and 3}: \ G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s) \tag{5.32}$$

Now, from Eq. (5.28) and its definitions, we form Δ and Δ_k. Hence,

$$\begin{aligned}\Delta = 1 &-[G_2(s)H_1(s) + G_4(s)H_2(s) + G_7(s)H_4(s) \\ &\qquad + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)] \\ &+[G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) \\ &\qquad + G_4(s)H_2(s)G_7(s)H_4(s)] \\ &-[G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]\end{aligned} \tag{5.33}$$

We form Δ_k by eliminating from Δ the loop gains that touch the kth forward path:

$$\Delta_1 = 1 - G_7(s)H_4(s) \tag{5.34}$$



Expressions (5.29), (5.33), and (5.34) are now substituted into Eq. (5.28), yielding the transfer function:

$$G(s) = \frac{T_1\Delta_1}{\Delta} = \frac{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)][1 - G_7(s)H_4(s)]}{\Delta} \tag{5.35}$$

Since there is only one forward path, $G(s)$ consists of only one term, rather than a sum of terms, each coming from a forward path.

Skill-Assessment Exercise 5.4

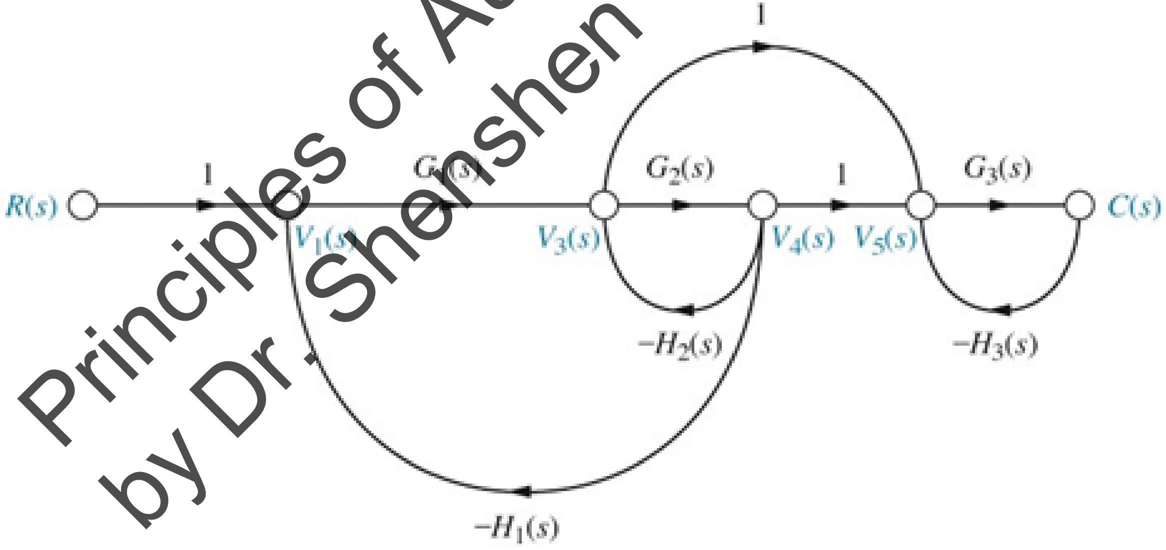
WileyPLUS
WPCS
Control Solutions

PROBLEM: Use Mason's rule to find the transfer function of the signal-flow diagram shown in Figure 5.19(*c*). Notice that this is the same system used in Example 5.2 to find the transfer function via block diagram reduction.

ANSWER:

$$T(s) = \frac{G_1(s)G_3(s)[1+G_2(s)]}{[1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)][1+G_3(s)H_3(s)]}$$

The complete solution is at www.wiley.com/college/nise.



5.4

Forward-path gains are $G_1G_2G_3$ and G_1G_3.
Loop gains are $-G_1G_2H_1$, $-G_2H_2$, and $-G_3H_3$.
Nontouching loops are $[-G_1G_2H_1][-G_3H_3] = G_1G_2G_3H_1H_3$ and $[-G_2H_2][-G_3H_3] = G_2G_3H_2H_3$.
Also, $\Delta = 1 + G_1G_2H_1 + G_2H_2 + G_3H_3 + G_1G_2G_3H_1H_3 + G_2G_3H_2H_3$.
Finally, $\Delta_1 = 1$ and $\Delta_2 = 1$.

Substituting these values into $T(s) = \dfrac{C(s)}{R(s)} = \dfrac{\sum_k T_k\Delta_k}{\Delta}$ yields

$$T(s) = \frac{G_1(s)G_3(s)[1 + G_2(s)]}{[1 + G_2(s)H_2(s) + G_1(s)G_2(s)H_1(s)][1 + G_3(s)H_3(s)]}$$

Summary

- The block diagram of a linear, time-invariant system consisted of four elements: *signals, systems, summing junctions, and pickoff points*;
- These elements were assembled into three basic forms: *cascade, parallel, and feedback*;
- Some basic operations were then derived: moving systems across summing junctions and across pickoff points;
- Once we recognized the basic forms and operations, we could reduce a complicated block diagram to a single transfer function relating input to output;

- Then we applied the methods of Topic 3 for analyzing and designing a second order system for transient behavior;
- The signal-flow representation of linear, time-invariant systems consists of two elements: nodes, which represent signals, and lines with arrows, which represent subsystems;
- *Mason's rule was used to derive the system's transfer function from the signal flow* graph.