09365060

Principles of Automatic Control (1)

自动控制原理1

Topic 2

Modeling in the Frequency Domain

(Chapter 2 in text book)

顾申申 Shenshen (Kevin) Gu

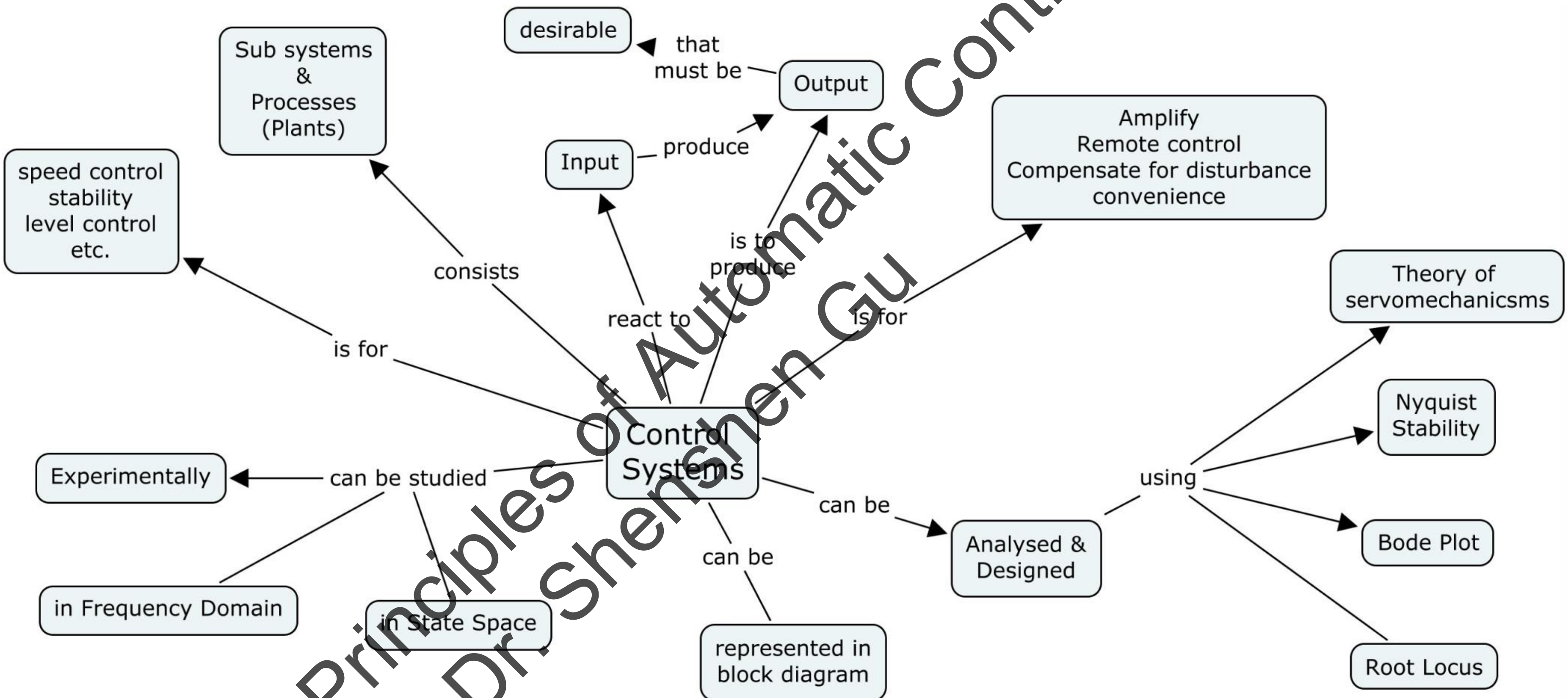
Ph.D. (CUHK), Associate Professor

Department of Automation

Shanghai University

gushenshen@shu.edu.cn

Review of Week 1



Learning Outcomes for Topic 2

After completing this topic, you will be able to:

- Find the Laplace transform of time functions and the inverse Laplace transform;
- Find the transfer function from a differential equation and solve the differential equation using the transfer function;
- Find the transfer function for some real linear, time-invariant systems.

Outline

- Brief Introduction
- Laplace Transform Review
- The Transfer Function
- Summary

New terminologies in this topic

- Complex 复数
- Imaginary 虚数
- Denominator 分母
- Numerator 分子
- Differential equation 微分方程
- Final value 终值
- Final value theorem 终值定理
- Laplace transform 拉普拉斯变换
- Inverse Laplace transform 拉普拉斯逆变换
- Partial Fraction Expansion 部分分式展开
- Linear system 线性系统
- Mathematical models 数学模型
- Transfer function 传递函数

Brief Introduction

- Control system is a dynamic system, so it can be represented by a differential equation.

$$\frac{d^n c(t)}{dt^n} + d_{n-1}\frac{d^{n-1} c(t)}{dt^{n-1}} + \cdots + d_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1}\frac{d^{m-1} r(t)}{dt^{m-1}} + \cdots + b_0 r(t)$$

- However representation in the time domain is not convenient and solving is also not easy (especially) when we deal with higher order systems. However, we still need to do this using physical laws *(This is our starting point)*

- ***Is it a good way to study control system by using differential equation?***

We would prefer a mathematical representation such that Input, Output, and system are distinct and separate.

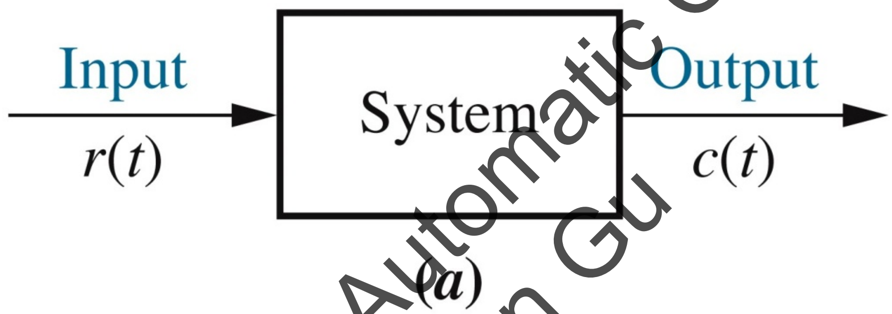
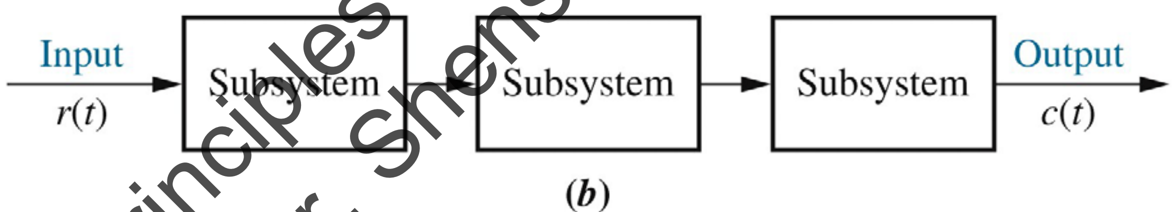


Figure 2.1a

Represent conveniently the interconnection of several sub-systems



Note: The input, $r(t)$, stands for *reference input*.
The output, $c(t)$, stands for *controlled variable*.

Figure 2.1b

Brief Introduction

- Alternate ways to represent the system, once we have the time domain relations, are:
  - a) **Frequency Domain representation**
    - We can transform the time domain relations into a model in the ***frequency domain*** ***(by Laplace Transform). The resulting model is easier to work with (only algebra is required).***
  - b) **State Space Model**
    - You can also translate the model in time domain into a *state space model* (*we will not do this*)

Laplace Transform Review: Definition

Definition

$$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}\,dt$$

Just one example.

Time function $f(t) = t\,u(t)$ for $t>0$

$$F(s) = \int_{0^-}^{\infty} t\,e^{-st}\,dt$$

$$= \int_{0^-}^{\infty} t \cdot \left[-\frac{d}{dt}\frac{1}{s}e^{-st}\right] dt$$

$$= -\frac{t}{s}e^{-st}\Big|_{0^-}^{\infty} + \int_{0^-}^{\infty} \frac{d(t)}{dt}\frac{1}{s}e^{-st}\,dt$$

$$= 0 + \int_{0^-}^{\infty} \frac{1}{s}e^{-st}\,dt = -\frac{1}{s^2}e^{-st}\Big|_{0^-}^{\infty}$$

$$= \frac{1}{s^2}$$

Definition of Inverse Transform

$$\mathscr{L}^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}\,ds = f(t)u(t)$$

$$u(t) = 1 \quad t>0$$
$$= 0 \quad t<0$$

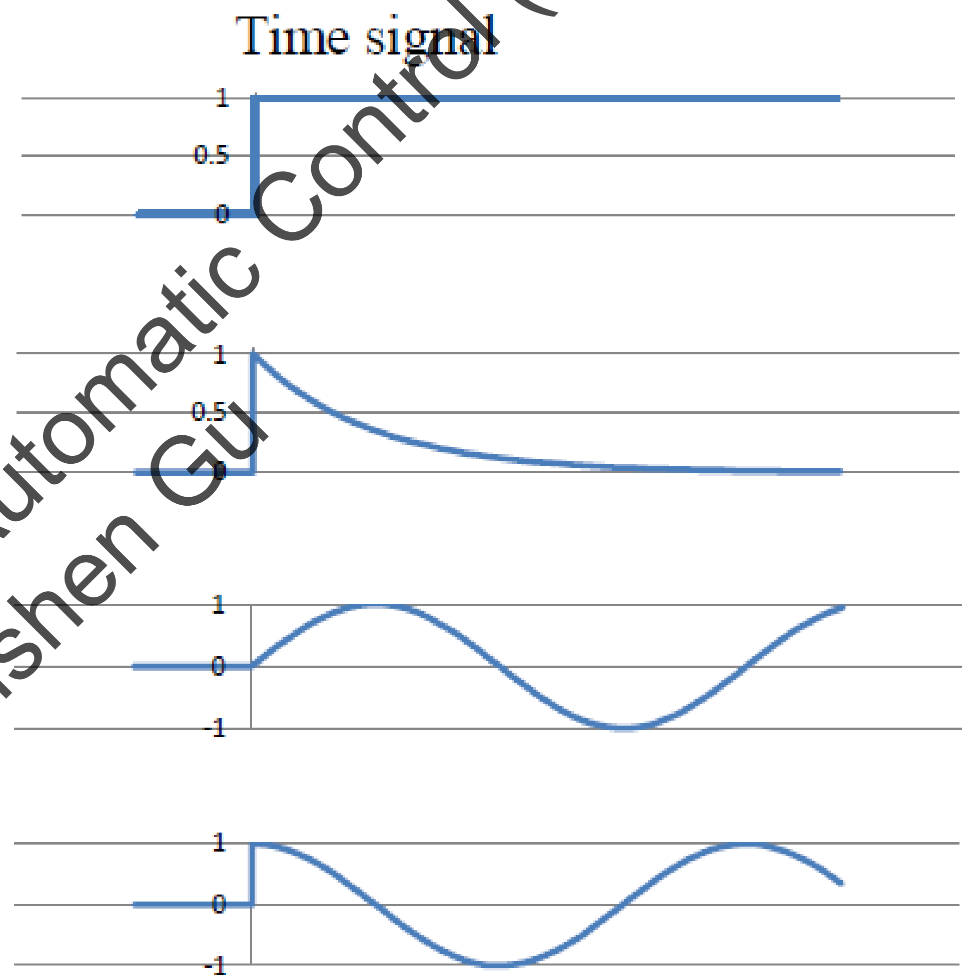
Laplace Transform Review: Laplace Transform of Signals

1. $u(t) \Leftrightarrow \frac{1}{s}$

2. $u(t)e^{-at} \Leftrightarrow \frac{1}{s+a}$

3. $u(t)\sin \omega t \Leftrightarrow \frac{\omega}{s^2+\omega^2}$

4. $u(t)\cos \omega t \Leftrightarrow \frac{s}{s^2+\omega^2}$



Laplace Transform Review: Laplace Transform Table

TABLE 2.1 Laplace transform table

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	~~$\frac{n!}{s^n + 1}$~~ $\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2 + \omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2 + \omega^2}$

Example 2.1

Laplace Transform of a Time Function

PROBLEM: Find the Laplace transform of $f(t) = Ae^{-at}u(t)$.

SOLUTION: Since the time function does not contain an impulse function, we can replace the lower limit of Eq. (2.1) with 0. Hence,

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt = \int_0^\infty Ae^{-at}e^{-st}\,dt = A\int_0^\infty e^{-(s+a)t}\,dt$$

$$= -\frac{A}{s+a}e^{-(s+a)t}\Big|_{t=0}^{\infty} = \frac{A}{s+a} \qquad (2.3)$$

Example 2.2

Inverse Laplace Transform

PROBLEM: Find the inverse Laplace transform of $F_1(s) = 1/(s+3)^2$.

SOLUTION: For this example we make use of the frequency shift theorem, Item 4 of Table 2.2, and the Laplace transform of $f(t) = tu(t)$, Item 3 of Table 2.1. If the inverse transform of $F(s) = 1/s^2$ is $tu(t)$, the inverse transform of $F(s+a) = 1/(s+a)^2$ is $e^{-at}tu(t)$. Hence, $f_1(t) = e^{-3t}tu(t)$.

$f(t)$	$F(s)$
$tu(t)$	$\frac{1}{s^2}$

$$L(tu(t)) = \frac{1}{s^2}$$

$$\mathscr{L}[e^{-at}f(t)] = F(s+a)$$

$$L\left(e^{-3t}tu(t)\right) = F(s+3) = \frac{1}{(s+3)^2}$$

Laplace Transform Review: Laplace Transform Theorems

TABLE 2.2 Laplace transform theorems

Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)] = F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s \to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s \to \infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.

[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Table 2.2

Laplace Transform Review: Linearity Property

$$\mathcal{L}[af_1(t)+bf_2(t)] = aF_1(s)+bF_2(s)$$

- Find the Laplace transform of:

$$y(t) = 5e^{-4t} + 7\cos(8t); \qquad t > 0$$

$$e^{-at} \Leftrightarrow \frac{1}{s+a} \qquad \cos\omega t \Leftrightarrow \frac{s}{s^2+\omega^2}$$

$$Y(s) = \frac{5}{s+4} + \frac{7s}{s^2+64}$$

Laplace Transform Review: Partial-Fraction Expansion

- Find the inverse Laplace transform of a complicated function -> Convert the function to a sum of simpler terms for which we know the Laplace transform of each term.
- This is called a partial-fraction expansion.

$$F(s)=\frac{N(s)}{D(s)}$$

- If the order of $N(s)<D(s)$ -> Partial-fraction expansion ✓
- If the order of $N(s)\geq D(s)$ -> Partial-fraction expansion ×
  - N(s) must be divided by D(s) successively until the result has a remainder whose numerator is of order less than its denominator -> Partial-fraction expansion ✓

- Example

$$F(s)=\frac{s^3+2s^2+6s+7}{s^2+s+5}$$

$$F(s)=s+1+\frac{2}{s^2+s+5}$$

$$f(t)=\frac{d\delta(t)}{dt}+\delta(t)+L^{-1}\left(\frac{2}{s^2+s+5}\right)$$

?

Using partial-fraction expansion.
Three cases:

Laplace Transform Review: Partial-Fraction Expansion
Case 1: Roots of the Denominator of *F(s)* are Real and Distinct

$$F(s)=\frac{2}{(s+1)(s+2)}$$

Real and distinct

- We can write the partial-fraction expansion as a sum of terms where each factor of the original denominator forms the denominator of each term, and constants, called *residues*, form the numerators.

$$F(s)=\frac{2}{(s+1)(s+2)}=\frac{K_1}{(s+1)}+\frac{K_2}{(s+2)}$$

$$\frac{2}{(s+1)(s+2)}\times(s+1)=\frac{K_1}{(s+1)}\times(s+1)+\frac{K_2}{(s+2)}\times(s+1)\Rightarrow\frac{2}{(s+2)}=K_1+\frac{K_2(s+1)}{(s+2)}$$

Let $s=-1$, we get $K_1=2$. Similarly, we can get $K_2=-2$

$$F(s)=\frac{2}{(s+1)}+\frac{-2}{(s+2)}\Rightarrow f(t)=\left(2e^{-t}-2e^{-2t}\right)u(t)$$

General form for case 1

$$F(s)=\frac{N(s)}{D(s)}=\frac{N(s)}{(s+p_1)(s+p_2)\cdots(s+p_m)\cdots(s+p_n)}$$

$$=\frac{K_1}{(s+p_1)}+\frac{K_2}{(s+p_2)}+\cdots+\frac{K_m}{(s+p_m)}+\cdots+\frac{K_n}{(s+p_n)}$$

- To evaluate each residue, K_i

$$F(s)\times(s+p_i)=\frac{N(s)}{(s+p_1)(s+p_2)\cdots(s+p_m)\cdots(s+p_n)}$$

$$=(s+p_i)\frac{K_1}{(s+p_1)}+(s+p_i)\frac{K_2}{(s+p_2)}+\cdots+(s+p_i)\frac{K_i}{(s+p_i)}+\cdots+(s+p_i)\frac{K_n}{(s+p_n)}$$

$$\left.\frac{\cancel{(s+p_i)}N(s)}{(s+p_1)(s+p_2)\cdots\cancel{(s+p_i)}\cdots(s+p_n)}\right|_{s=-p_i}=K_i$$

Example 2.3

Laplace Transform Solution of a Differential Equation

PROBLEM: Given the following differential equation, solve for $y(t)$ if all initial conditions are zero. Use the Laplace transform.

$$\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = 32u(t) \tag{2.14}$$

SOLUTION: Substitute the corresponding $F(s)$ for each term in Eq. (2.14), using Item 2 in Table 2.1, Items 7 and 8 in Table 2.2, and the initial conditions of $y(t)$ and $dy(t)/dt$ given by $y(0-) = 0$ and $\dot{y}(0-) = 0$, respectively. Hence, the Laplace transform of Eq. (2.14) is

$$s^2Y(s) + 12sY(s) + 32Y(s) = \frac{32}{s} \tag{2.15}$$

Solving for the response, $Y(s)$, yields

$$Y(s) = \frac{32}{s(s^2 + 12s + 32)} = \frac{32}{s(s+4)(s+8)} \tag{2.16}$$

To solve for $y(t)$, we notice that Eq. (2.16) does not match any of the terms in Table 2.1. Thus, we form the partial-fraction expansion of the right-hand term and match each of the resulting terms with $F(s)$ in Table 2.1. Therefore,

$$Y(s) = \frac{32}{s(s+4)(s+8)} = \frac{K_1}{s} + \frac{K_2}{(s+4)} + \frac{K_3}{(s+8)} \tag{2.17}$$

where, from Eq. (2.13),

$$K_1 = \frac{32}{(s+4)(s+8)}\bigg|_{s\to 0} = 1 \tag{2.18a}$$

$$K_2 = \frac{32}{s(s+8)}\bigg|_{s\to -4} = -2 \tag{2.18b}$$

$$K_3 = \frac{32}{s(s+4)}\bigg|_{s\to -8} = 1 \tag{2.18c}$$

Hence,

$$Y(s) = \frac{1}{s} - \frac{2}{(s+4)} + \frac{1}{(s+8)} \tag{2.19}$$

Since each of the three component parts of Eq. (2.19) is represented as an $F(s)$ in Table 2.1, $y(t)$ is the sum of the inverse Laplace transforms of each term. Hence,

$$y(t) = (1 - 2e^{-4t} + e^{-8t})u(t) \tag{2.20}$$

Laplace Transform Review: Partial-Fraction Expansion
Case 2: Roots of the Denominator of *F(s)* are Real and Repeated

$$F(s)=\frac{2}{(s+1)(s+2)^2}$$

Real and distinct　　　　Real and repeated

- We can write the partial-fraction expansion as a sum of terms, where each factor of the denominator forms the denominator of each term. In addition, each multiple root generates additional terms consisting of denominator factors of reduced multiplicity.

$$F(s)=\frac{2}{(s+1)(s+2)^2}=\frac{K_1}{(s+1)}+\frac{K_2}{(s+2)^2}+\frac{K_3}{(s+2)}$$

$K_1=2$ can be determined with the method described in case 1

$$\frac{2}{(s+1)(s+2)^2}\times(s+2)^2=\frac{K_1}{(s+1)}(s+2)^2+\frac{K_2}{(s+2)^2}(s+2)^2+\frac{K_3}{(s+2)}(s+2)^2$$

Let $s=-2$, we get $K_2=-2$. And differentiate the above equation with respect to s, we get:

$$\frac{-2}{(s+1)^2}=\frac{(s+2)s}{(s+1)^2}K_1+K_3 \Rightarrow K_3=-2,\ f(t)=2e^{-t}-2te^{-2t}+2e^{-2t}$$

General form for case 2

$$F(s)=\frac{N(s)}{D(s)}$$

$$=\frac{N(s)}{(s+p_1)^r(s+p_2)\cdots(s+p_n)}$$

$$=\frac{K_1}{(s+p_1)^r}+\frac{K_2}{(s+p_1)^{r-1}}+\cdots+\frac{K_r}{(s+p_1)}$$

$$+\frac{K_{r+1}}{(s+p_2)}+\cdots+\frac{K_n}{(s+p_n)}$$

$$F_1(s)=(s+p_1)^r F(s)$$

$$=\frac{(s+p_1)^r N(s)}{(s+p_1)^r(s+p_2)\cdots(s+p_n)}$$

$$=K_1+(s+p_1)K_2+(s+p_1)^2K_3+\cdots+(s+p_1)^{r-1}K_r$$

$$+\frac{K_{r+1}(s+p_1)^r}{(s+p_2)}+\cdots+\frac{K_n(s+p_1)^r}{(s+p_n)}$$

K_1 can be determined by setting $s=-p_1$

$$K_i=\frac{1}{(i-1)!}\frac{d^{i-1}F_1(s)}{ds^{i-1}}\bigg|_{s\to -p_i}\qquad i=1,2,\ldots,r;0!=1$$

Laplace Transform Review: Partial-Fraction Expansion
Case 3: Roots of the Denominator of *F(s)* are Complex or Imaginary

$$F(s)=\frac{3}{s\left(s^2+2s+5\right)}$$

Complex

This function can be expanded in the following form:

$$\frac{3}{s\left(s^2+2s+5\right)}=\frac{K_1}{s}+\frac{K_2 s+K_3}{s^2+2s+5}$$

K_1 is found in the usual way to be *3/5*. K_2 and K_3 can be found by first multiplying the above equation by the lowest common denominator, $s(s^2+2s+5)$, and clearing the fractions. After simplification with $K_1=3/5$, we obtain

$$3=\left(K_2+\frac{3}{5}\right)s^2+\left(K_3+\frac{6}{5}\right)s+3$$

$$K_2=-\frac{3}{5},K_3=-\frac{6}{5},F(s)=\frac{3}{s\left(s^2+2s+5\right)}=\frac{3/5}{s}-\frac{3}{5}\frac{s+2}{s^2+2s+5}$$

The last term can be shown to be the sum of the Laplace transforms of an exponentially damped sine and cosine

$$L\left(Ae^{-at}\cos\omega t\right)=\frac{A(s+a)}{(s+a)^2+\omega^2}$$

$$L\left(Be^{-at}\sin\omega t\right)=\frac{B\omega}{(s+a)^2+\omega^2}$$

$$L\left(Ae^{-at}\cos\omega t+Be^{-at}\sin\omega t\right)=\frac{A(s+a)+B\omega}{(s+a)^2+\omega^2}$$

$$F(s)=\frac{3/5}{s}-\frac{3}{5}\frac{(s+1)+(1/2)2}{(s+1)^2+2^2}$$

$$f(t)=\frac{3}{5}-\frac{3}{5}e^{-t}\left(\cos 2t+\frac{1}{2}\sin 2t\right)=0.6-0.671e^{-t}\cos(2t-\phi),\phi=26.57^\circ$$

f(t) is a constant plus an exponentially damped sinusoid

General form for case 3

$$F(s)=\frac{N(s)}{D(s)}=\frac{N(s)}{(s+p_1)(s^2+as+b)\cdots}$$

$$=\frac{K_1}{(s+p_1)}+\frac{(K_2s+K_3)}{(s^2+as+b)}+\cdots$$

← Complex or imaginary

The Transfer Function

Considering a general nth-order, linear, time-invariant differential equation:

$$a_n \frac{d^n c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} c(t)}{dt^{n-1}} + \cdots + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \cdots + b_0 r(t)$$

where $c(t)$ is the output, $r(t)$ is the input, and the a_i's, b_i's, and the form of the differential equation represent the system.

Taking the Laplace transform of both side

$$a_n s^n C(s) + a_{n-1} s^{n-1} C(s) + \cdots + a_0 C(s) + \text{initial condition terms involving } c(t)$$
$$= b_m s^m R(s) + b_{m-1} s^{m-1} R(s) + \cdots + b_0 R(s) + \text{initial condition terms involving } r(t)$$

Assuming that ***all initial conditions are zero***

$$\left(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0\right) C(s) = \left(b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0\right) R(s)$$

The Transfer Function

Form the ratio of the output, *C(s)*, divided by the input transform, *R(s)*

$$\frac{C(s)}{R(s)} = G(s) = \frac{\left(b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0\right)}{\left(a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0\right)}$$

We call this ratio, *G(s)*, the *transfer function*

The transfer function can be represented as a *block diagram*

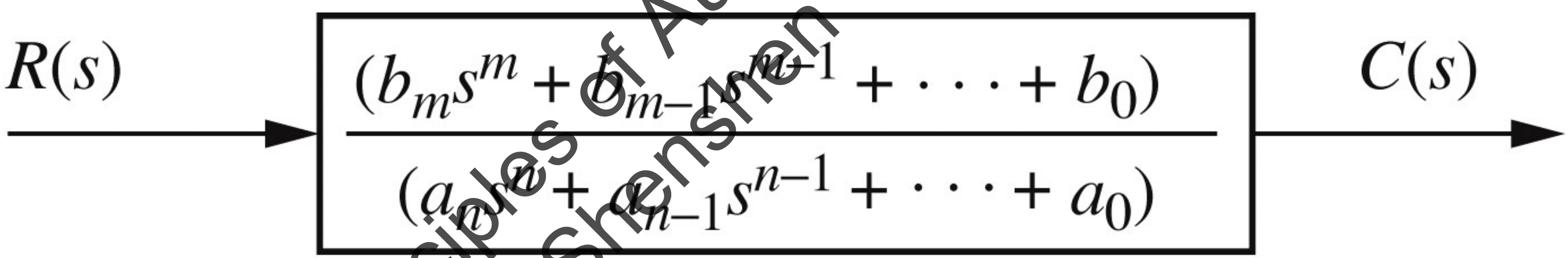


Figure 2.2

We can find the output, *C(s)* by using $C(s) = R(s)G(s)$

The Transfer Function: Examples

Example 2.4

Transfer Function for a Differential Equation

PROBLEM: Find the transfer function represented by

$$\frac{dc(t)}{dt} + 2c(t) = r(t) \tag{2.55}$$

SOLUTION: Taking the Laplace transform of both sides, assuming zero initial conditions, we have

$$sC(s) + 2C(s) = R(s) \tag{2.56}$$

The transfer function, $G(s)$, is

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2} \tag{2.57}$$

The Transfer Function: Examples

Example 2.5

TryIt 2.6

Use the following MATLAB and Symbolic Math Toolbox statements to help you get Eq. (2.60).

```
syms s
C=1/(s*(s+2))
C=ilaplace(C)
```

TryIt 2.7

Use the following MATLAB statements to plot Eq. (2.60) for t from 0 to 1 s at intervals of 0.01 s.

```
t=0:0.01:1;
plot...
 (t,(1/2-1/2*exp(-2*t)))
```

System Response from the Transfer Function

PROBLEM: Use the result of Example 2.4 to find the response, $c(t)$ to an input, $r(t) = u(t)$, a unit step, assuming zero initial conditions.

SOLUTION: To solve the problem, we use Eq. (2.54), where $G(s) = 1/(s+2)$ as found in Example 2.4. Since $r(t) = u(t)$, $R(s) = 1/s$, from Table 2.1. Since the initial conditions are zero,

$$C(s) = R(s)G(s) = \frac{1}{s(s+2)} \tag{2.58}$$

Expanding by partial fractions, we get

$$C(s) = \frac{1/2}{s} - \frac{1/2}{s+2} \tag{2.59}$$

Finally, taking the inverse Laplace transform of each term yields

$$c(t) = \frac{1}{2} - \frac{1}{2}e^{-2t} \tag{2.60}$$

Skill-Assessment Exercises

Skill-Assessment Exercise 2.3

PROBLEM: Find the transfer function, $G(s) = C(s)/R(s)$, corresponding to the differential equation $\frac{d^3c}{dt^3} + 3\frac{d^2c}{dt^2} + 7\frac{dc}{dt} + 5c = \frac{d^2r}{dt^2} + 4\frac{dr}{dt} + 3r$.

ANSWER: $G(s) = \frac{C(s)}{R(s)} = \frac{s^2 + 4s + 3}{s^3 + 3s^2 + 7s + 5}$

The complete solution is at www.wiley.com/college/nise.

Skill-Assessment Exercise 2.4

PROBLEM: Find the differential equation corresponding to the transfer function,

$$G(s) = \frac{2s + 1}{s^2 + 6s + 2}$$

ANSWER: $\frac{d^2c}{dt^2} + 6\frac{dc}{dt} + 2c = 2\frac{dr}{dt} + r$

The complete solution is at www.wiley.com/college/nise.

Skill-Assessment Exercises

Skill-Assessment Exercise 2.5

PROBLEM: Find the ramp response for a system whose transfer function is

$$G(s) = \frac{s}{(s+4)(s+8)}$$

ANSWER: $c(t) = \frac{1}{32} - \frac{1}{16}e^{-4t} + \frac{1}{32}e^{-8t}$

The complete solution is at www.wiley.com/college/nise.

WileyPLUS

WPCS

Control Solutions

Skill-Assessment Exercises: Solutions

Skill-Assessment Exercise 2.3

PROBLEM: Find the transfer function, $G(s) = C(s)/R(s)$, corresponding to the differential equation $\frac{d^3c}{dt^3} + 3\frac{d^2c}{dt^2} + 7\frac{dc}{dt} + 5c = \frac{d^2r}{dt^2} + 4\frac{dr}{dt} + 3r$.

ANSWER: $G(s) = \frac{C(s)}{R(s)} = \frac{s^2 + 4s + 3}{s^3 + 3s^2 + 7s + 5}$

The complete solution is at www.wiley.com/college/nise.

Taking the Laplace transform of the differential equation assuming zero initial conditions yields:

$$s^3C(s) + 3s^2C(s) + 7sC(s) + 5C(s) = s^2R(s) + 4sR(s) + 3R(s)$$

Collecting terms,

$$(s^3 + 3s^2 + 7s + 5)C(s) = (s^2 + 4s + 3)R(s)$$

Thus,

$$\frac{C(s)}{R(s)} = \frac{s^2 + 4s + 3}{s^3 + 3s^2 + 7s + 5}$$

Skill-Assessment Exercises: Solutions

Skill-Assessment Exercise 2.4

PROBLEM: Find the differential equation corresponding to the transfer function,

$$G(s) = \frac{2s+1}{s^2+6s+2}$$

ANSWER: $\frac{d^2c}{dt^2} + 6\frac{dc}{dt} + 2c = 2\frac{dr}{dt} + r$

The complete solution is at www.wiley.com/college/nise.

$$G(s) = \frac{C(s)}{R(s)} = \frac{2s+1}{s^2+6s+2}$$

Cross multiplying yields,

$$\frac{d^2c}{dt^2} + 6\frac{dc}{dt} + 2c = 2\frac{dr}{dt} + r$$

Skill-Assessment Exercises: Solutions

Skill-Assessment Exercise 2.5

PROBLEM: Find the ramp response for a system whose transfer function is

$$G(s) = \frac{s}{(s+4)(s+8)}$$

ANSWER: $c(t) = \frac{1}{32} - \frac{1}{16}e^{-4t} + \frac{1}{32}e^{-8t}$

The complete solution is at www.wiley.com/college/nise.

WileyPLUS
WPCS
Control Solutions

$$C(s) = R(s)G(s) = \frac{1}{s^2} * \frac{s}{(s+4)(s+8)} = \frac{1}{s(s+4)(s+8)} = \frac{A}{s} + \frac{B}{(s+4)} + \frac{C}{(s+8)}$$

where

$$A = \left.\frac{1}{(s+4)(s+8)}\right|_{S\to 0} = \frac{1}{32},\ B = \left.\frac{1}{s(s+8)}\right|_{S\to -4} = -\frac{1}{16},\ \text{and } C = \left.\frac{1}{s(s+4)}\right|_{S\to -8} = \frac{1}{32}$$

Thus,

$$c(t) = \frac{1}{32} - \frac{1}{16}e^{-4t} + \frac{1}{32}e^{-8t}$$

Summary

- We discussed how to find a mathematical model, called a *transfer function*, for linear, time-invariant systems;
- The transfer function is defined as $G(s)=C(s)/R(s)$, or the ratio of the Laplace transform of the output to the Laplace transform of the input;
- This relationship is algebraic and also adapts itself to modeling interconnected subsystems;
- Now that we have our transfer function, we can evaluate its response to a specified input in the next topic.