# 09365060

# Principle of Automatic Control (2)

# 自动控制原理2 (全英语教学课程)

## Solution to Home Work Part 2

(Chapter 13 in text book)

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# 13-3(a)

- For each F(z), find f(kT) using partial-fraction expansion.

**a.** $F(z)=\dfrac{z(z+3)(z+5)}{(z-0.4)(z-0.6)(z-0.8)}$

$$F(z)=\frac{z(z+3)(z+5)}{(z-0.4)(z-0.6)(z-0.8)}$$

$$\frac{F(z)}{z}=\frac{229.5}{z-0.4}-\frac{504}{z-0.6}+\frac{275.5}{z-0.8}$$

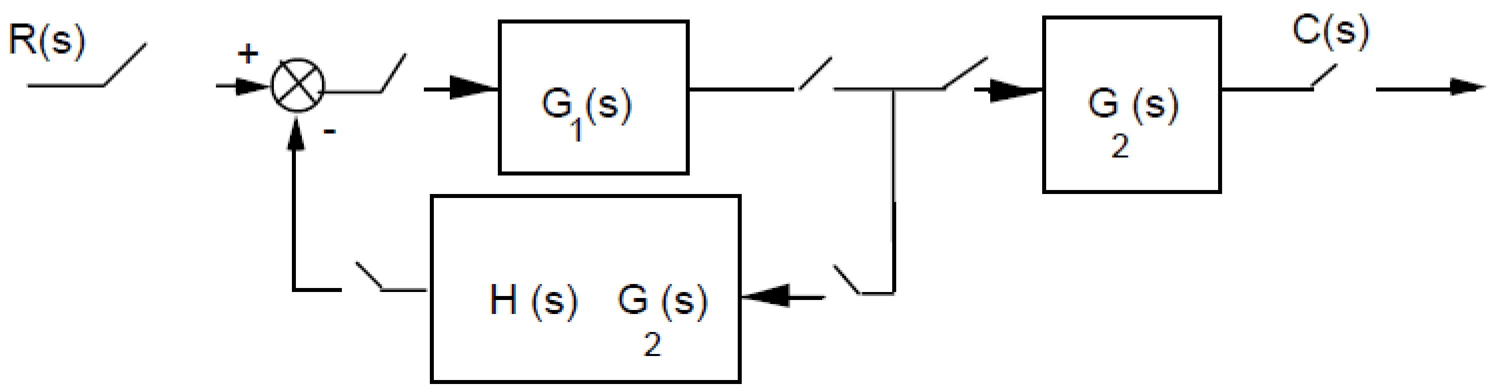
$$F(z)=\frac{229.5z}{z-0.4}-\frac{504z}{z-0.6}+\frac{275.5z}{z-0.8}$$

$$f(kT)=229.5(0.4)^k-504(0.6)^k+275.5(0.8)^k,\ k=0,\ 1,\ 2,\ 3,\ldots$$

# 13-9(a)

- Find T(z)=C(z)/R(z) for each of the systems shown in Figure P13.2.

**a.** Add phantom samplers at the input, output, and feedback path after H(s). Push $G_2$ (s) and its input sampler to the right past the pickoff point. Add a phantom sampler after $G_1(s)$. Hence,




From this block diagram, $T(z) = \dfrac{G_1(z)G_2(z)}{1 + G_1(z)HG_2(z)}$ .

# 13-14

- Find the range of gain, K, to make the system shown in Figure P13.7 stable.

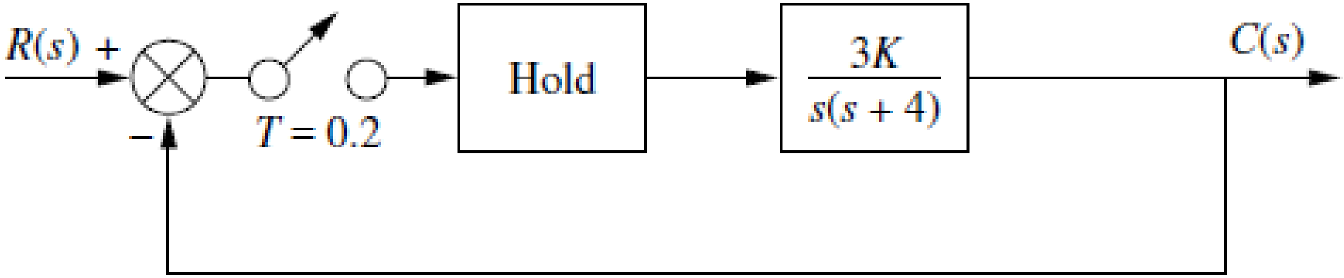



FIGURE P13.7

Since $H(s)=1$, the z-transform of the closed-loop system, $T(z)=\dfrac{G(z)}{1+G(z)}$

To find $G(z)$, first find the partial-fraction expansion of $G(s)$.

$$\because\ G(s)=K(1-e^{-Ts})\frac{3}{s^2(s+4)}=K(1-e^{-Ts})\left(\frac{A}{s}+\frac{B}{s+4}+\frac{C}{s^2}\right)$$

$$\therefore\ As^2+4As+Bs^2+Cs+4C=(A+B)s^2+(4A+C)s+4C=3$$

$$\therefore\ \begin{cases} A+B=0 \\ 4A+C=0 \\ 4C=3 \end{cases} \Rightarrow \begin{cases} A=-0.1875 \\ B=0.1875 \\ C=0.75 \end{cases}$$

$$\therefore\ G(s)=K(1-e^{-Ts})\left(-\frac{0.1875}{s}+\frac{0.1875}{s+4}+\frac{0.75}{s^2}\right)$$

Taking the z-transform, T=0.2, we obtain

$$\therefore G(z)=\frac{K(z-1)}{z}\left[-0.1875\frac{z}{z-1}+0.1875\frac{z}{z-e^{-4T}}+0.75\frac{Tz}{(z-1)^2}\right]$$

$$=K\left[-0.1875+0.1875\frac{z-1}{z-e^{-4\times(0.2)}}+0.75\frac{0.2}{z-1}\right]$$

$$=K\left[-0.1875+0.1875\frac{z-1}{z-e^{-0.8}}+\frac{0.15}{z-1}\right]$$

$$\therefore G(z)=K\frac{(0.0467z+0.0359)}{(z-0.4493)(z-1)}=K\frac{0.0467(z+0.7687)}{(z-0.4493)(z-1)}$$

$$\therefore T(z)=\frac{G(s)}{1+G(s)}=\frac{K0.0467(z+0.7687)}{K0.0467(z+0.7687)+(z-0.4493)(z-1)}$$

The closed-loop characteristic equation

$$z^2 + (0.0467K - 1.4493)z + (0.0359K + 0.4493) = 0$$

$$\because \; z = \frac{s+1}{s-1}$$

$$\therefore \left(\frac{s+1}{s-1}\right)^2 + (0.0467K - 1.4493)\left(\frac{s+1}{s-1}\right) + (0.0359K + 0.4493) = 0$$

$$s^2 + 2s + 1 + (0.0467K - 1.4493)(s^2 - 1) + (0.0359K + 0.4493)(s^2 - 2s + 1) = 0$$

$$s^2 + 2s + 1 + 0.0467Ks^2 - 0.0467K - 1.4493s^2 + 1.4493 + 0.0359Ks^2 - 0.0718Ks + 0.0359K + 0.4493s^2$$

$$- 0.8986s + 0.4493 = 0$$

$$(0.0826K)s^2 + (-0.0718K + 1.1014)s + (-0.0108K + 2.8986) = 0$$

The Routh table is shown below.

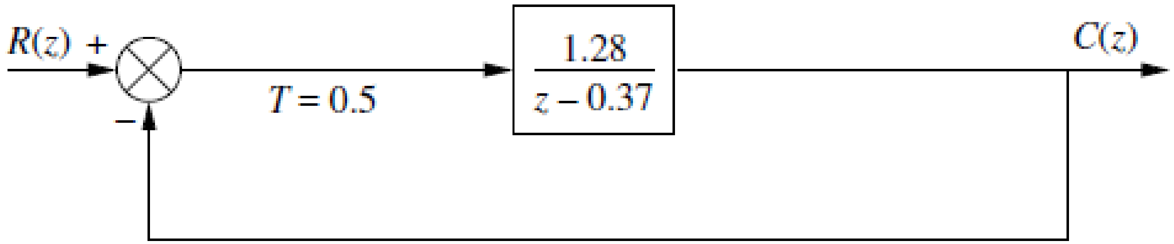
| | | |
|---|---|---|
| $s^2$ | 0.0826K | -0.0108K+2.8986 |
| $s^1$ | -0.0718K+1.1014 | 0 |
| $s^0$ | -0.0108K+2.8986 | |

$$\begin{cases} 0.0826K > 0 \\ -0.0718K + 1.1014 > 0 \\ -0.0108K + 2.8986 > 0 \end{cases} \Rightarrow \begin{cases} K > 0 \\ K < 15.34 \\ K < 268.39 \end{cases}$$

∴ The range of gain K is $0 < K < 15.34$ ,when the system is stable.

# 13-15(System 3)

- Find the static error constants and the steady-state error for each of the digital systems shown in Figure P13.8 if the inputs are
- a. u(t)
- b. tu(t)
- c. $0.5t^2u(t)$




System 3

$$G_z = \frac{1.28}{z-0.37}$$

First, test stability .

$$T_z = \frac{G_z}{1+G_z}$$

$$T_z = 1.28\frac{1}{z+0.91}$$

The system is stable. The closed-loop pole is inside the unit circle. Now find the static error constants and the steady-state errors.

$$K_p = \lim_{z\to 1} G(z) = 2.03 \qquad e^*(\infty) = \frac{1}{1+K_p} = 0.33$$

$$K_v = \frac{1}{T}\lim_{z\to 1}(z-1)G(z) = 0 \qquad e^*(\infty) = \frac{1}{K_v} = \infty$$

$$K_a = \frac{1}{T^2}\lim_{z\to 1}(z-1)^2 G(z) = 0 \qquad e^*(\infty) = \frac{1}{K_a} = \infty$$