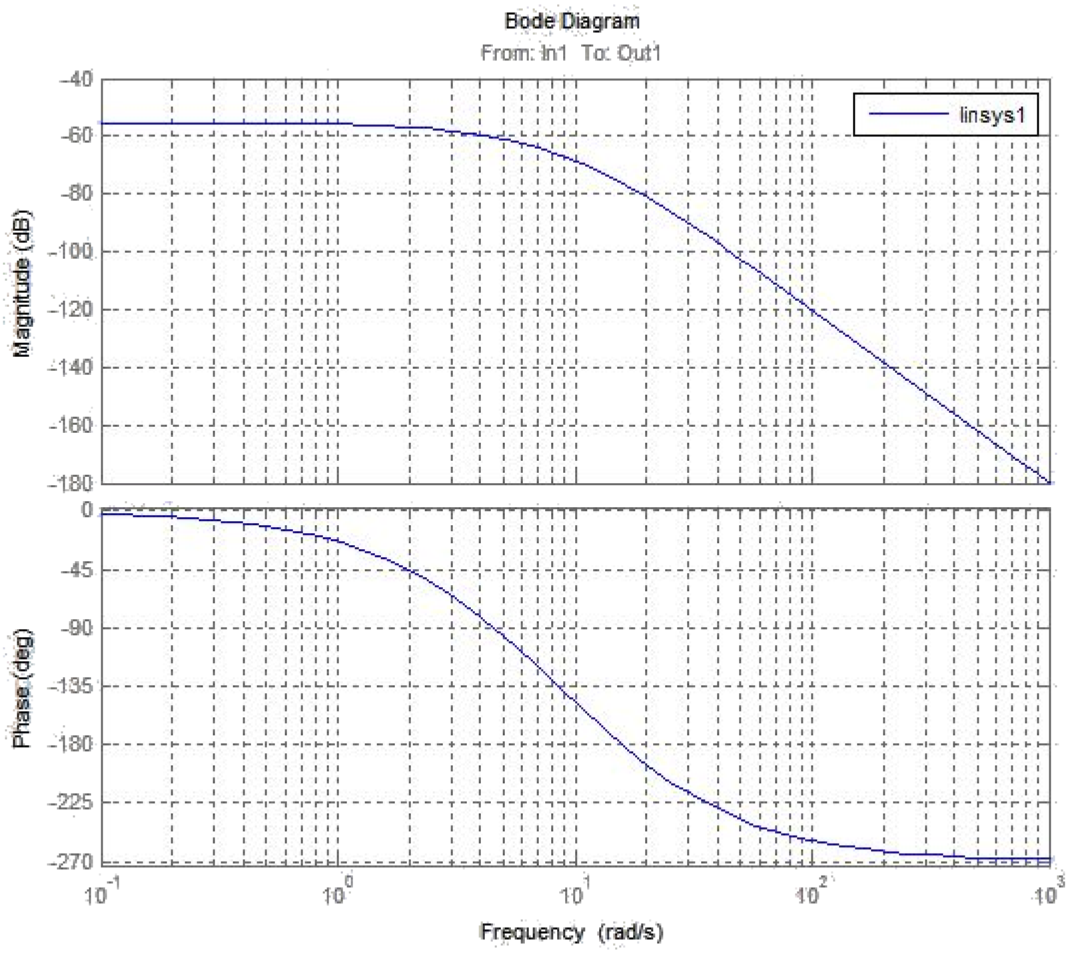
11.2 For the system $G(s) = \frac{K}{(s+4)(s+10)(s+15)}$ **,design the gain, K, for a phase margin of 40° .**

Solution:

The Bode plot for K=1 is shown below.



A phase margin of 40° , which is obtained when the phase angle = -180° + 40° =-140° .

This phase angle occurs at $\omega = 9.12 rad/s$ .The magnitude curve at this frequency is -67.5dB.

K can be raised from its current value of unity to $K = 10^{\frac{67.5}{20}} = 2371.37$ .So K=2371.37 will meet a 40° phase margin.

11.8 Design a lag compensator so that the system of Figure P11.1 where

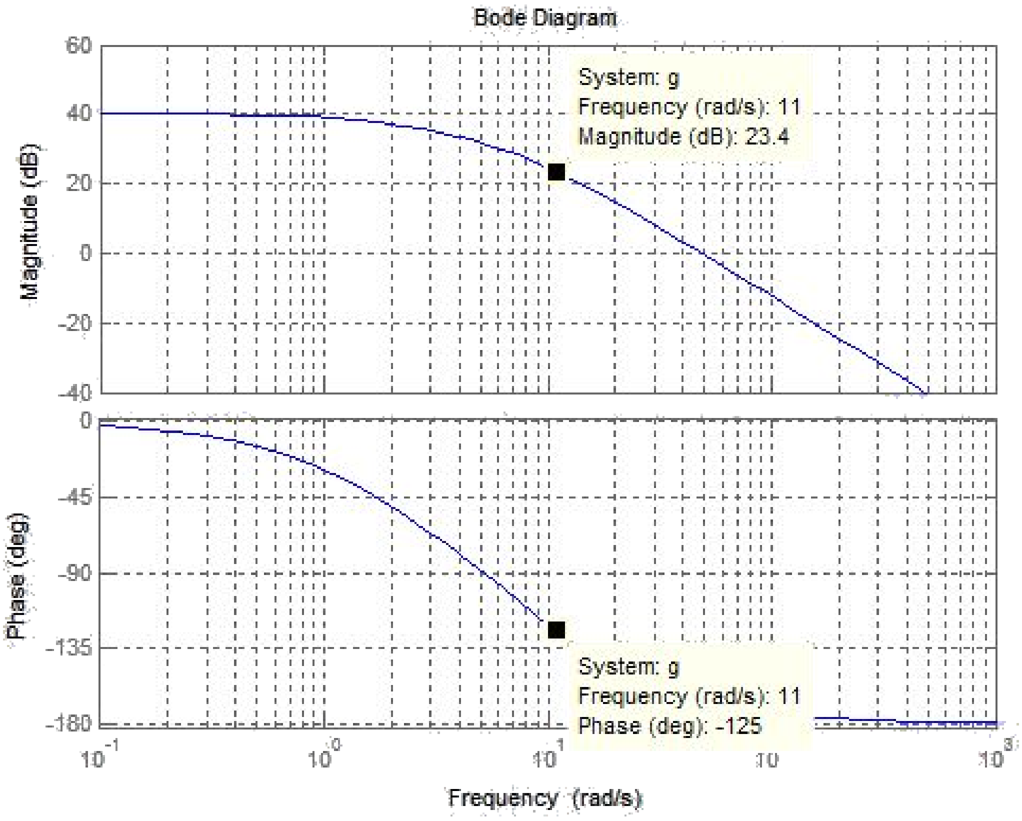
$$G(s)=\frac{K(s+4)}{(s+2)(s+6)(s+8)}$$

operates with a 45° phase margin and a static error constant of 100.

Solution:

$$K_P=\lim_{s\to 0}G(s)=\frac{4K}{2\times 6\times 8}=100\Rightarrow K=2400$$

The Bode plot for K=2400 is shown below.



A phase margin is 45° .Adding 10° to compensate for the phase angle contribution of the lag, we use 55° .Thus, we look for a phase angle of -180° +55° = -125° .

The frequency at which this phase occurs is 11 rad/s. At this frequency, the magnitude must go through 0dB.Presently,the magnitude plot is 23.4dB.Therefore draw the high frequency asymptote of the lag compensator at -23.4dB.Insert a break at 1.1rad/s. At this frequency, draw -20dB/dec slope until it intersects 0dB.The low frequency break is 0.075 rad/s. $\beta=\frac{0.075}{1.1}=0.068$

Hence, $G_{lag}(s)=0.068\frac{(s+1.1)}{(s+0.075)}$

The compensated system's forward transfer function is thus

$$G(s)G_{lag}(s) = \frac{163.2(s+4)(s+1.1)}{(s+2)(s+6)(s+8)(s+0.075)}$$

11.19 Using frequency response methods to design a lag-lead compensator for a unity feedback system where

$$G(s) = \frac{K(s+7)}{s(s+5)(s+15)}$$

and the following specifications are to be met: percent overshoot=15%,settling time=0.1 second, and K_v=1000.

Solution:

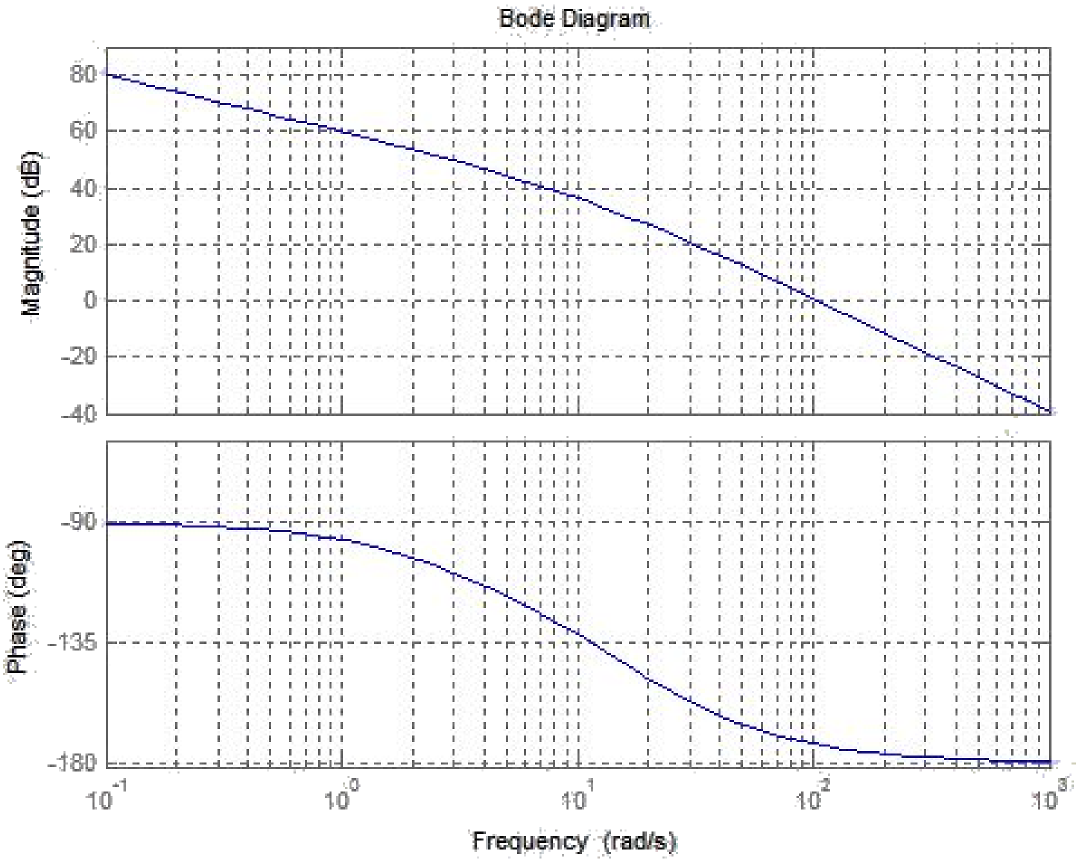
A 15% overshoot requires $\zeta = \frac{-\ln\left(\frac{15}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{15}{100}\right)}} = 0.517$ .The required bandwidth is then calculated as

$$\omega_{BW} = \frac{4}{T_s\zeta}\sqrt{\left(1-2\zeta^2\right)+\sqrt{4\zeta^4-4\zeta^2+2}}$$
$$= \frac{4}{0.1\times0.517}\sqrt{\left(1-2\times0.517^2\right)+\sqrt{4\times0.517^4-4\times0.517^2+2}}$$
$$= 96.9 rad/s$$

In order to meet the steady-state error requirement of $K_v = \lim_{s\to0} sG(s) = \frac{7K}{5\times15} = 1000$

∴K=10714.

The Bode plot for K=10714 is shown below.



$$\phi_M = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}} = \tan^{-1}\frac{2\times 0.517}{\sqrt{-2\times 0.517^2+\sqrt{1+4\times 0.517^4}}} = 53.18^\circ$$

Adding a 5° correction factor, the required phase margin is 58.18° .

Let us select $\omega = 0.8\omega_{BW} = 0.8\times 96.9 = 77.52 rad/s$.

At the new phase-margin frequency ,the phase angle is-170° . Thus, the phase margin is 180° -170° =10° ,The lead compensator must contribute $\phi_{\max} = 58.18^\circ - 10^\circ = 48.18^\circ$

$$\because \phi_{\max} = \sin^{-1}\frac{1-\beta}{1+\beta}, \beta = \frac{1-\sin\phi_{\max}}{1+\sin\phi_{\max}} = \frac{1-\sin(48.18^\circ)}{1-\sin(48.18^\circ)} = 0.146$$

①We now design the lag compensator：Its higher break frequency is $\frac{1}{T_2} = 7.752$.The lag compensator's pole is $\frac{\beta}{T_2} = 0.146\times 7.752 = 1.132$

Finally, the lag compensator's gain is $\beta = 0.146 \therefore G_{lag}(s) = \beta \frac{\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{\beta}{T_2}\right)} = 0.146 \frac{s + 7.752}{s + 1.132}$

②Then we design the lead compensator: The lead zero is

$\frac{1}{T_1} = \omega \times \sqrt{\beta} = 77.52 \times \sqrt{0.146} = 29.62$ Also, $\frac{1}{\beta T_1} = \frac{1}{0.146} \times 29.62 = 202.88$

Finally $\gamma = \frac{1}{\beta} = 6.85 \therefore G_{lead}(s) = \gamma \frac{\left(s + \frac{1}{T_1}\right)}{\left(s + \frac{\gamma}{T_1}\right)} = 6.85 \frac{s + 29.62}{s + 202.88}$

In summary, $G_{lag}(s) = 0.146 \frac{s + 7.752}{s + 1.132}$; $G_{lead}(s) = 6.85 \frac{s + 29.62}{s + 202.88}$;K=10714

Thus, the lag-lead-compensated forward path is

$$G_{lag-lead-comp}(s) = \frac{10715.0714(s+7)(s+7.752)(s+29.62)}{s(s+5)(s+15)(s+1.132)(s+202.88)}$$