**8.**

**a.** Pole: -2; $c(t) = A + Be^{-2t}$ ; first-order response.

**b.** Poles: -3, -6; $c(t) = A + Be^{-3t} + Ce^{-6t}$; overdamped response.

**c.** Poles: -10, -20; Zero: -7; $c(t) = A + Be^{-10t} + Ce^{-20t}$; overdamped response.

**d.** Poles: $(-3+j3\sqrt{15})$, $(-3-j3\sqrt{15})$ ; $c(t) = A + Be^{-3t}\cos(3\sqrt{15}\ t + \phi)$; underdamped.

**e.** Poles: j3, -j3; Zero: -2; $c(t) = A + B\cos(3t + \phi)$; undamped.

**f.** Poles: -10, -10; Zero: -5; $c(t) = A + Be^{-10t} + Cte^{-10t}$; critically damped.

**15.**

$$C(s) = \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_n^2 - \zeta^2\omega_n^2}$$

$$= \frac{1}{s} - \frac{(s + \zeta\omega_n) + \zeta\omega_n}{(s+\zeta\omega_n)^2 + (\omega_n\sqrt{1 - \zeta^2})^2} = \frac{1}{s} - \frac{(s+\zeta\omega_n) + \frac{\zeta\omega_n}{\omega_n\sqrt{1 - \zeta^2}}\omega_n\sqrt{1 - \zeta^2}}{(s+\zeta\omega_n)^2 + (\omega_n\sqrt{1 - \zeta^2})^2}$$

Hence, $c(t) = 1 - e^{-\zeta\omega_n t}\left(\cos \omega_n\sqrt{1 - \zeta^2}t + \frac{\zeta}{\sqrt{1 - \zeta^2}}\sin \omega_n\sqrt{1 - \zeta^2}t\right)$

$$= 1 - e^{-\zeta\omega_n t}\sqrt{1 + \frac{\zeta^2}{1-\zeta^2}}\cos(\omega_n\sqrt{1 - \zeta^2}\ t - \phi) = 1 - e^{-\zeta\omega_n t}\frac{1}{\sqrt{1-\zeta^2}}\cos(\omega_n\sqrt{1 - \zeta^2}\ t - \phi),$$

where $\phi = \tan^{-1}\frac{\zeta}{\sqrt{1 - \zeta^2}}$

**20.**

**a.** $\omega_n^2 = 16$ r/s, $2\zeta\omega_n = 3$. Therefore $\zeta = 0.375$, $\omega_n = 4$. $T_s = \frac{4}{\zeta\omega_n} = 2.667$ s; $T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} =$ 0.8472 s; $\%OS = e^{-\zeta\pi / \sqrt{1 - \zeta^2}} \times 100 = 28.06\ \%$; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1) = 1.4238$; therefore, $T_r = 0.356$ s.

**b.** $\omega_n^2 = 0.04$ r/s, $2\zeta\omega_n = 0.02$. Therefore $\zeta = 0.05$, $\omega_n = 0.2$. $T_s = \frac{4}{\zeta\omega_n} = 400$ s; $T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} =$ 15.73 s; $\%OS = e^{-\zeta\pi / \sqrt{1 - \zeta^2}} \times 100 = 85.45\ \%$; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)$; therefore, $T_r = 5.26$ s.

**c.** $\omega_n^2 = 1.05 \times 10^7$ r/s, $2\zeta\omega_n = 1.6 \times 10^3$. Therefore $\zeta = 0.247$, $\omega_n = 3240$. $T_s = \frac{4}{\zeta\omega_n} = 0.005$ s; $T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.001$ s; $\%OS = e^{-\zeta\pi / \sqrt{1 - \zeta^2}} \times 100 = 44.92\ \%$; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)$; therefore, $T_r = 3.88\text{x}10^{-4}$ s.

**23.**

**a.** $\zeta = \dfrac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.56$, $\omega_n = \dfrac{4}{\zeta T_s} = 35.71$. Therefore, poles $= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$= -20 \pm j29.59$.

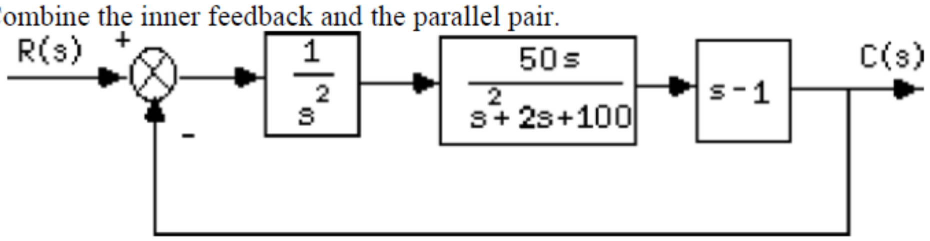
**b.** $\zeta = \dfrac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.517$, $\omega_n = \dfrac{\pi}{T_P\sqrt{1-\zeta^2}} = 0.734$.

Therefore, poles $= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -0.3794 \pm j0.6283$.

**c.** $\zeta\omega_n = \dfrac{4}{T_s} = 0.571$, $\omega_n\sqrt{1-\zeta^2} = \dfrac{\pi}{T_p} = 1.5708$ Therefore, poles $= -0.571 \pm j1.5708$.

**1.**

**a.** Combine the inner feedback and the parallel pair.

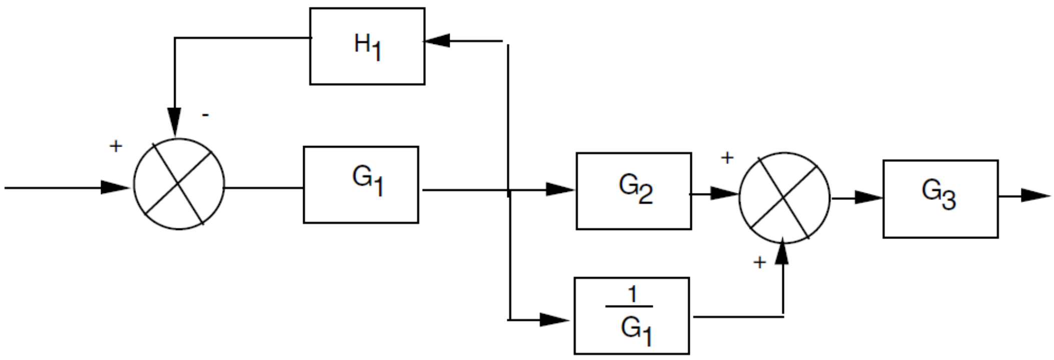



Multiply the blocks in the forward path and apply the feedback formula to get,

$$T(s) = \frac{50(s-2)}{s^3 + 2s^2 + 150s - 50}$$

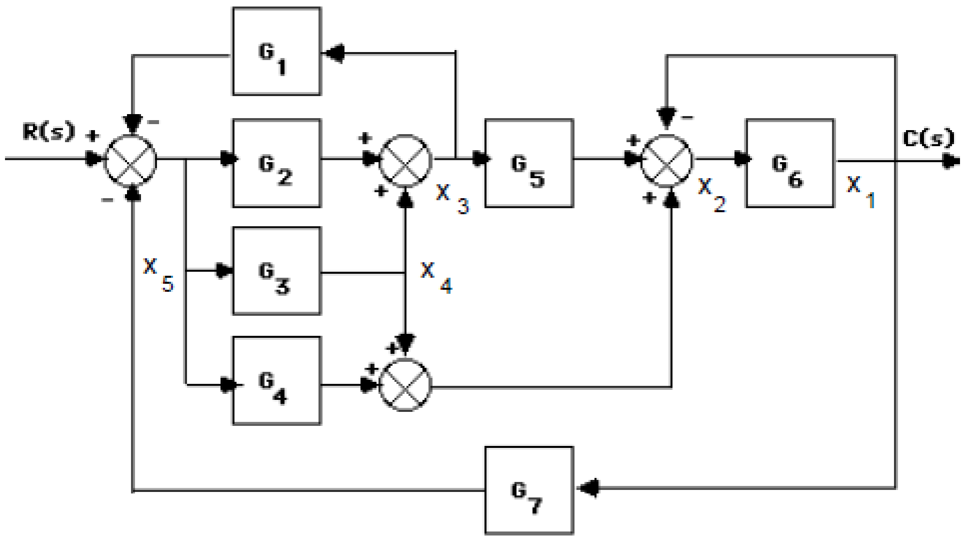
**2.**

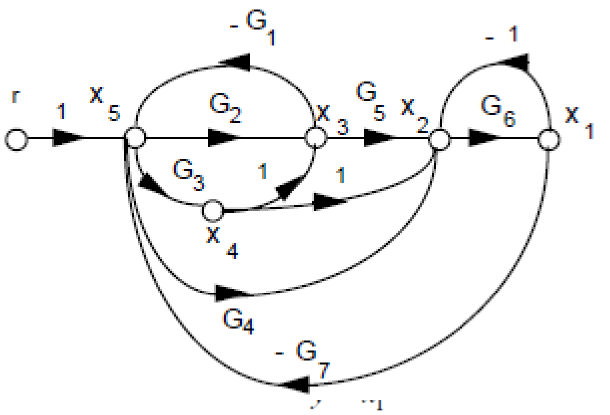
Push $G_1(s)$ to the left past the pickoff point.




Thus, $T(s) = \left(\dfrac{G_1}{1+G_1H_1}\right)\left(G_2 + \dfrac{1}{G_1}\right)G_3 = \dfrac{(G_1G_2+1)G_3}{(1+G_1H_1)}$

b.




**26.**

$\Delta = 1 + [G_2G_3G_4 + G_3G_4 + G_4 + 2] + [G_3G_4 + G_4]$; $T_1 = G_1G_2G_3G_4$; $\Delta_1 = 1$. Therefore,

$$T(s) = \frac{T_1\Delta_1}{\Delta} = \frac{G_1G_2G_3G_4}{3 + G_2G_3G_4 + 2G_3G_4 + 2G_4}$$